The Arithmetic Teacher

APRIL • 1959

Differentiating Arithmetic Instruction
GEORGE H. McMEEN

Research in Arithmetic Instruction—1958
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THE ARITHMETIC TEACHER

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Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editors: MARGUERITE BRYDEGAARD, San Diego State College, San Diego, Calif.; E. GLENADINE GIBB, State Teachers College, Cedar Falls, Iowa; JOHN R. CLARK, New Hope, Pa.; JOSEPH J. URBANCEK, Chicago Teachers College, Chicago 21, Ill.

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THE ARITHMETIC TEACHER

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Differentiating Arithmetic Instruction for Various Levels of Achievement

GEORGE H. MCMEEN

Newark State College, Union, N. J.

A GREAT DEAL has been said and written concerning provisions for differentiating arithmetic instruction for various levels of achievement and yet probably most teachers, including the writer, feel that there is much to be learned about this topic. This article will attempt to clarify and evaluate various procedures for differentiating arithmetic instruction currently in use or being tested in many of our schools today. The writer makes no pretense of possessing final answers or ready-made prescriptions which always work in any situation for existing research warrants no such conclusions. Much depends upon the nature of the arithmetic program, its administration, the pupils, and the capabilities of the teacher. Procedures which work for one teacher may not be equally effective for another. It is hoped that the reader will acquire a few new ideas from this article and be stimulated to extend his efforts in this critical but often sadly neglected area.

The topic under consideration is particularly timely because of our present emphasis upon modernizing our mathematics program and providing pupils with the best possible opportunities for securing a strong mathematical background. The rapidly changing secondary mathematics program presents new challenges to the elementary mathematics program to put its house in order so that the transition from one program to the other will be gradual and continuous. One of the great challenges facing us at the moment is to do a better job with the gifted child who has often been sadly neglected in many of our schools. Indeed the future of our nation may very well hinge upon the job the teachers of America do today in properly preparing, guiding, and encouraging our gifted children toward careers of leadership and research vital to the national welfare. It is well known that likes and dislikes toward mathematics, dispositions toward study and experimentation, feelings of confidence or insecurity, and so forth, are molded in the elementary school and that the pattern so cast is not easily changed thereafter.

Nature of Differentiation

Clarification of what the topic under discussion means should be helpful before proceeding further. Differentiating arithmetic instruction refers to adjustments in depth and scope of arithmetic topics presented in keeping with individual and trait differenences. Individual differences refer to the variations that exist among various individuals of a group on any single measure or trait. Trait differences refer to the variations in different abilities possessed by an individual. Pupils differ one from another in any

single arithmetic ability and any individual has differing arithmetic abilities with respect to the various component parts of arithmetic. Since provisions for differentiating arithmetic instruction usually accompany procedures for grouping pupils, although they are by no means identical, the writer feels it necessary to provide a resume of procedures for grouping pupils for arithmetic instruction.

Procedures for Grouping Pupils

Considerable experience at the elementary school level leads the writer to conclude that most teachers utilizing three ability groupings for reading instruction have but a single class grouping for arithmetic instruction. This is unfortunate for individual differences are fully as pronounced in arithmetic as in reading and it seems logical to conclude that many children at either end of the arithmetic spectrum are not receiving the attention due them. Inasmuch as there is a high correlation between arithmetic and reading abilities it is unfortunate that more teachers do not as a minimum adjustment utilize their reading ability groupings for arithmetic instruction on many occasions.

What are the reasons why arithmetic instruction has not received the attention accorded to reading? Why have many teachers never attempted ability grouping or differentiation of instruction in arithmetic? Some of the reasons are: (1) Until recently arithmetic has been regarded chiefly as a tool or skill subject as contrasted with a system of thought and a respected branch of mathematics. (2) Mathematics has often been looked upon with fear and distrust by teachers who shied away from it themselves and knew little of its role in society. Developments of the last year paving the way into a space age have at least had the effect of increasing the respect of laymen and teachers toward the importance of modern mathematics. (3) Whereas in reading it is fairly simple to surround children with good books it is difficult to do this in arithmetic because of a lack of good books in sufficient quantity written for differing ability levels. Present arithmetic texts are written for the average child with only token provisions for the slow or rapid learner. (4) Teachers have found from experience that it takes a great deal of time, effort, and ability to operate more than one arithmetic grouping and consequently have shied away from several groupings because of the work involved an insufficient amount of time allotted to the arithmetic program which would make grouping possible. (5) Many of our teachers know only the skill aspect of arithmetic and are incompetent to direct rapid learners into a fuller knowledge of the subject. It would be easily possible to add to this list of reasons why many teachers have never seriously ventured into the realm of differentiating arithmetic instruction but there would be little constructive value in doing so.

Out of many experiments in grouping children for arithmetic instruction have emerged a number of generally accepted principles worthy of consideration: (1) It is never possible by any method of grouping to have a completely homogeneous group of pupils. We may narrow the range of abilities but we do not eliminate them. Consequently every teacher should be keenly aware of individual differences regardless of how homogeneous her group may be. The writer once queried a group of teachers with the question, "How many groups should you have in your arithmetic class?" An experienced teacher replied, "As many as there are pupils in the class." This is a good reply for it evidences concern for the individual pupil. (2) It is never possible, try as we may, to bring the performance level of the slow-moving child up to the level of the rapid learner unless we place the latter in an intellectual deep-freeze. By allowing rapid learners to turn to other tasks as soon as they have completed minimum arithmetic assignments many teachers have in effect put the rapid learner in an arithmetic straight-jacket. (3) The range of abilities in any subject increases year by year as children move through the elementary school. Hence the need for differentiating arithmetic instruction grows

more acute at each succeeding grade level. (4) Most authorities agree that there should be little if any differentiation of arithmetic topics studied in the first eight grades, particularly so in the first six grades. It is generally conceded that a study of whole numbers, fractions, and decimals represents a common core needed by all normal citizens. Consequently differentiation at this level should be confined to levels of learning, depth, and scope of treatment, the goal being that each child is challenged to do his best at his ability level. Pupils should be expected neither to over-perform or underperform. (5) The cooperation of the pupil in any differentiated program is essential and it is important that the pupil take pride in accomplishing goals he is capable of achieving and has helped to formulate. (6) Any system of grouping or differentiation of instruction should be flexible. There is probably no single criterion which will divide pupils perfectly according to ability. Hence any system of grouping will involve mistakes which will need to be corrected in light of subsequent evidence. When pupils improve or fall behind both they and the teacher should be willing to make changes. Flexibility helps to alleviate the stigma associated with being permanently assigned to a slow group. (7) Grouping alone is no panacea for handling individual differences. It would be a grave mistake to provide no differentiation of instruction among differing ability groupings. Rapid learners need much less work with manipulative materials and drill than do slow learners. They are able to move more quickly to the abstract level of performance and to greater depth. Rapid learners are generally capable of much more self-direction and sustained effort. Individual or group study of assigned topics for investigation in the library is a common type of differentiation for rapid learners even in classrooms having no system of grouping. Such activity is valuable as it is does not obviate the need for competent instruction in mathematical meanings and the structure of arithmetic. All too often rapid learners sidestep this important phase of the arithmetic program usually because they are not properly challenged and directed by the teacher or the arithmetic text.

Programs for Individual Differences

Programs for providing for individual differences in arithmetic through some form of grouping fall into one of two broad classifications: (1) The first plan centers about organizing the school program itself through homogeneous grouping of students, assignment of teachers according to teaching speciality or preference, and adjustments within the schedule. (2) The second plan in contrast emphasizes the values associated with heterogeneous grouping of children with the teacher utilizing special teaching methods for caring for individual differences within the groupings. Let us briefly consider the characteristics of each of these programs, their advantages and limitations.

The first plan of administratively grouping pupils homogeneously although widely practiced at the junior and senior high levels has won comparatively little acceptance in the elementary school. Educators whose training was steeped in an interpretation of Dewey's philosophy have usually interpreted the elementary school as a place for "cooperative group living" and hence have decried suggestions for homogeneous grouping on the grounds that is undemocratic and is likely to lead to a class-structured society. Today many educators are questioning this narrow interpretation. Experience in grouping gifted children together have convinced many educators that smugness and aloofness are not characteristic traits of gifted children when homogeneous grouping is properly practiced. Indeed these traits are more likely to develop when gifted children are forced to move at the level of slow- or average-moving pupils than when they move at their own pace in the company of others of comparable ability. Many favorable indications for homogeneous grouping are beginning to appear when new instructional methods and materials are used with gifted children. So far research in homogeneous grouping has

been directed toward finding what benefits may accrue for the gifted child. A current research experiment seeks to determine among other things what effect the removal of gifted children from the classroom has upon the remaining children.[1]

One of the simplest administrative adjustments in ability grouping which should be practiced more in my opinion is for the elementary school principal having two or three classes at a given grade level to divide them according to ability and then assign teachers to the classes who are competent to teach them. The pupils in the top group need a teacher who can properly challenge and direct them and little good is likely to ensue from such grouping without such a teacher.

A second plan schedules pupils at a given grade level for one-half day for reading, language arts, and social studies under a homeroom teacher proficient in these subjects. For the remaining half day the pupils are homogeneously grouped on a non-graded basis under teaching specialists for arithmetic and the other subjects. This plan is now being tested experimentally in the schools of Ossing and Long Beach, N. Y. under the direction of Dean George Stoddard of New York University. A variation of this plan used in a few large cities sends gifted children from their normal classroom situation to a special school for the gifted for a half day for specialized instruction on a non-graded basis in such subjects as mathematics and science.

A third plan which has few advocates is to completely departmentalize the elementary school under teaching specialists for all subjects. Advocates of this proposal argue that many elementary teachers are not sufficiently grounded in the various subjects to do justice to them, particularly so in the case of arithmetic. They usually point out that many European schools have operated profitably in this fashion for many years and that it is high time for our schools to adopt their pattern. Critics of this proposal are quick to point out that we educate all of our youth as opposed to the selected few in many European schools, that children at this tender age need the molding influence of one teacher who knows them well, and that the real goals of education at this level lie not in the acquisition of knowledge alone but in good citizenship training and personality development. Obviously rhetoric in either direction accomplishes little and research alone can confirm the values for or against homogeneous grouping.

[1] Talented Youth Project, Horace Mann-Lincoln Institute of School Experimentation, Teachers College, Columbia University, New York City. Current Research Projects (unpublished).

Plan for a Regular Grade

Let us now direct our attention toward provisions for handling individual differences in normal teaching situations where teachers work with heterogeneously composed groups of children. These teachers have found from experience that some sort of grouping or differentiation is essential in working with children of widely differing abilities and backgrounds. Some teachers utilize the same groupings for arithmetic as for reading. Others form their arithmetic groups according to arithmetic test scores plus other criteria. A suggestion to the teacher inexperienced in arithmetic grouping is to first form two ability groupings and learn to handle them successfully. After mastering this situation the teacher should then try three groupings. The usual range of abilities in most classrooms indicates the need for a group for the rapid learners, a group for the near-average learners, and a group for the slow learners.

Sound practice indicates the advisability of maintaining flexible groupings with no child permanently assigned to any one group. Pupils should be consulted in choosing a group in keeping with their needs. There are many opportunities in the arithmetic program for the teacher to bring all of the pupils together so that they may profit from the experiences of the others. Particularly is this true when a new topic is being introduced and pupils are challenged to discover the concepts underlying the topic. However once these have been discovered,

varying rates of learning quickly begin to manifest themselves and the wise teacher returns to her separate groupings so that she can better differentiate her arithmetic program.

As previously indicated many problems beset the teacher who tackles ability grouping in arithmetic and differentiation of instruction. One of these problems is the problem of scheduling. My early experience teaching the upper four grades in one room of a two-room rural school has convinced me that at least a minimum of 15 minutes is needed to conduct any arithmetic lesson worthy of the name with more time highly desirable. Consequently if three groupings are scheduled and 15 minutes is spent with each group daily, a minimum of 45 minutes is needed for the total arithmetic period. According to this plan the schedule might read:

TIME	RAPID LEARNERS	AVERAGE LEARNERS	SLOW LEARNERS
9:00–9:15	Individual study	Group lesson	Individual study
9:15–9:30	Individual study	Individual study	Group lesson
9:30–9:45	Group lesson	Individual study	Individual study

Since slow learners lack the capacity for sustained study it is wise to schedule their group lesson during the central interval as shown. Many teachers have found that 15 minute lessons are too brief and not having sufficient time to extend their program, to, say, three 20-minute lessons prefer to have no more than two group lessons during a single day. This of course means that with three groups, one group will have no lesson with the teacher for that day.

Any plan for ability grouping or differentiation of instruction requires careful planning by the teacher. Assignments must be made clearly so that pupils can work as independently as possible. As long as routine skills are involved this is relatively simple but when mathematical meanings are properly stressed along with other enrichments of the arithmetic program, the teacher's task is far from simple. Frequently the teacher may find it necessary to use as many as three different texts of differing grade levels with her ability groups. Keeping the pupils somewhat together on a common topic such as the addition of whole numbers at three different levels is difficult and yet this is far more desirable that having pupils work on completely unrelated topics at the same time.

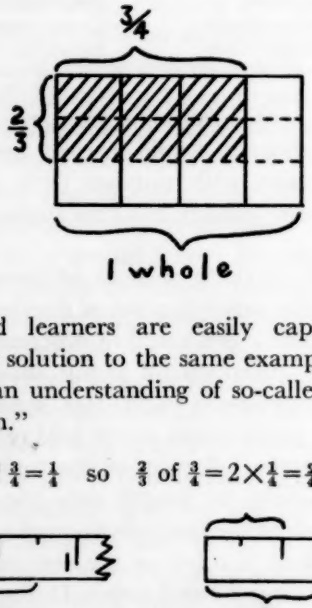
Let us now turn our attention to provisions for differentiating arithmetic instruction. This becomes the crux of providing for individual differences regardless of what method of grouping is practiced. I have previously indicated that differentiation in elementary arithmetic should be principally in depth and scope of topics taught with all pupils studying essentially the same topics. In the teaching of certain fundamentals such as number facts it is difficult to differentiate but even here some adjustments can be provided. Slow learners need many more experiences with manipulative materials than rapid learners who can move quickly to mental procedures for regrouping numbers in the abstract. Rapid learners need fewer drill lessons with number facts and can move more quickly to social applications of them.

In the teaching of more advanced topics such as the multiplication of fractions in the sixth grade a great deal of differentiation is possible. Although all pupils should have some experiences with this topic, slow-moving pupils might not be held responsible for mastering all cases of the topic. These pupils might be taught only the simpler cases such as the multiplication of a fraction by a whole number or the case of a unit-fraction times a unit-fraction. Differentiation of this kind is referred to as *differentiation in scope*. If further learning of more advanced cases dictates mechanical learning it is questionable whether greater scope should be attempted, particularly when we consider the few social applications of the topic the child is likely to encounter. The remaining children should be taught all cases of the

topic with the rapid learners being challenged to provide diagrammatic and mathematical explanations of the various operations. This is what we mean by *differentiation in depth.* It is in the critical area of mathematical understandings that the teacher who does not know arithmetic well herself is apt to let rapid learners by-pass this important phase of the program and concentrate chiefly upon the acquisition of skills.

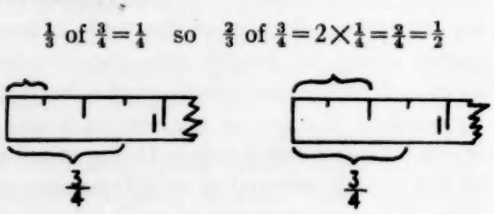
Differentiation in Depth

Possibilities for differentiation in depth occur in such examples as $\frac{2}{3}$ of $\frac{3}{4}$. Most average learners are capable of understanding or being led to discover how the diagram below shows that $\frac{2}{3}$ of $\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$. This leads to the generalization that multiplying the numerators together supplies the number of new parts and that multiplying the denominators together supplies the name or size of the new parts.



Rapid learners are easily capable of another solution to the same example leading to an understanding of so-called "cancellation."

$\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$ so $\frac{2}{3}$ of $\frac{3}{4}=2\times\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$



From these discoveries the rapid learner can be led to to see that the same result can be obtained more quickly by cancellation as shown on the right. The generalization here is that since we make the factor on the right one-third as large by dividing the numerator by 3, we must compensate by making the factor on the left 3 times as large. Similarly we can make the factor on the left one-half as large provided we make the factor on the right twice as large, thus completing the cancellation.

$$\frac{\cancel{2}}{1}\ \text{of}\ \frac{1}{\cancel{4}}$$

Rolland Smith[2] has written an excellent article on provisions for individual differences. He recommends the practice in presenting a new topic for the first time of first giving a bird's-eye view of the whole topic on a small scale to all pupils at the same time. This plan is beneficial at all levels of mathematics teaching. When we develop a topic piecemeal the learner often gets lost in the maze of operations involved and never quite sees the relation between the various parts of the picture. Smith illustrates his point in the case of decimal fractions and shows how the teacher can quickly develop some of the background of decimals, the way they are read and written, and the application of the four fundamental processes to them in simple cases. He states that such a preview need not take place in a single lesson but may require three or four lessons. Following the preview differentiation should follow at individual or group levels since detailed development of any single phase of the topic will proceed at varying ability rates.

Three Groups in Grade Four

Finally, let us briefly consider a theoretical fourth grade lesson in compound subtraction of three-place numbers to see how the teacher might differentiate her instruction. Let us assume that all of the pupils were exposed to a meaningful development of the topic in the third grade and that some telescoped reteaching of the topic has been done near the beginning of the fourth

[2] Rolland R. Smith, "Provisions for Individual Differences," *The Learning of Mathematics; Its Theory and Practice*, pp. 286–290. Twenty-First Yearbook of the National Council of Teachers of Mathematics. Washington, D.C.: The National Council of Teachers of Mathematics, 1953.

grade. At the time of this particular lesson let us assume that such a wide range of abilities exist on the topic that the teacher finds it advisable to divide her pupils into three ability groups. What might be the differentiated activities of these groups for the day?

Slow learners: Their assignment might include only one phase of the topic such as simply changing a ten to 10 ones or a hundred to 10 tens but not both at the same time. Their assignment might be to develop once again the rationalization given in their third-grade text with a large amount of time spent in manipulating objective materials such as play money, bundled sticks, or cardboard markers. The group lesson for these pupils should make ample use of pupil and teacher demonstrations of the principles involved.

Average learners: Their assignment should embody the development in their fourth-grade text. These children are probably ready for the inclusion of both changing a ten to 10 ones and a hundred to 10 tens with perhaps some examples involving double changing in both the tens and hundreds positions. Many of the children in this group will profit by making at least some use of manipulative materials although to a lesser extent than the slow learners. Positional markers in a place-pocket chart will probably suffice for the average learner since by this time he has learned that a single marker in hundreds place stands for a collection of 100 ones or 10 tens. Pupils in this group will need much less attention in their assignment than in the slow-moving group. The teacher should be alert in the group lesson to see that these pupils really understand what they are doing and that they are not merely subtracting mechanically. Most of these pupils should be at the developmental stage where they are ready to discard the crutch whereas the slow learners have great need of it still.

Rapid learners: These pupils probably have no need for manipulative materials and can proceed immediately to an abstract representation of the meaning of subtraction, writing out an example or two in terms of place-value words, e.g. hundreds, tens, and ones. These pupils should be challenged to discover other ways of performing the subtraction such as subtracting the hundreds first, then the tens, and then the ones. Historical methods of subtraction might be introduced at the discretion of the teacher after the pupils firmly understand their own method. The pupils might profitably be challenged to find a quicker way of subtracting in an example such as 400−158. This might lead to the discovery that 400 contains 40 tens and that if one of these tens is changed to 10 ones, 39 tens are left. At a still higher grade level compound subtraction in different number bases may be profitably introduced. Rapid learners might also be assigned the task of formulating problems in compound subtraction originating from some unit of study in the classroom. As pointed out previously, rapid learners are often insufficiently challenged to gain deeper mathematical insights, conditioned usually by the teacher not possessing them herself. Only determination and continued study by the teacher can rectify this situation.

Let us now consider still another kind of differentiation; vertical differentiation. So far the types discussed have been chiefly horizontal in nature with rapid learners being exposed to a much broader and deeper program than slow learners. Vertical differentiation refers to the inclusion of a greater number of topics at a given grade level than those generally studied. Three types of vertical differentiation in arithmetic are:

(1) Probably the most common but certainly questionable method is for the teacher to simply introduce more advanced topics into her program than those normally presented in her text or course outline. This is a dangerous procedure if done on a hit or miss basis without conferring with teachers who will teach the pupils next or keeping careful records of the topics taught. It has been my observation that teachers presenting advanced topics at a given grade level usually introduce them mechanically as an expedient to keep rapid learners busy. Probably nothing is more likely to quench the

initiative of a rapid learner than later to have to sit patiently in a class while the teacher explains operations he already knows how to perform. Even though the rapid learner was exposed only to a mechanical explanation of how to get the answer initially, he is now apt to take little interest in a meaningful presentation because the keen edge of learning has been removed from the situation. When this form of vertical differentiation is practiced with some pupils and not others, the problem of record keeping is a major issue.

(2) A second plan of vertical differentiation consists of simply having a rapidly moving group move ahead or "skip" a grade. If this is done, it is imperative that the teacher below and the one above the grade skipped cooperate in seeing that the pupils cover all of the essential topics otherwise omitted. Teachers should never assume that simply because rapid learners possess the ability, that they will of their own initiative study and master all of the topics skipped. The writer can testify to this point having been skipped through the 8B as a child with resulting deficiencies in subsequent work.

(3) A third and more desirable method of vertical differentiation consists of completely reorganizing the arithmetic program for rapid learners into a six- or seven-year program, usually the latter. Where this is done, algebra is usually begun in what is normally the eighth grade. The problem here is not so much whether rapid learners can theoretically learn sufficient arithmetic in six or seven years as it is to find suitable materials and teachers for such programs. It is possible to use existing texts and move more rapidly through them, omitting certain topics and practice materials. However this places great responsibility upon the teacher in judging what may be deleted without injury to the program. Departing from the sequence of the text series demands much experience and good judgment. Authors spend long hours building their arithmetic program and the teacher who thinks she can do so on the spur of the moment is apt to lead her pupils into unforeseen difficulties. Ideally we need new arithmetic text series for rapid learners organized on a six- or seven-year program. To the best of my knowledge no company has thus far been willing to assume the financial risk of publishing such a series because of the prospects of limited sales. It is likely that most schools experimenting with six- or seven-year arithmetic programs are accomplishing their purpose chiefly by deleting heavily from among the topics normally studied in the seventh and eighth grades. Such an adjustment is obviously inferior to a program of reorganization starting at the beginning of the school program.

Without doubt we know less today about successful procedures for differentiating vertically than according to any other plan. Unless a carefully developed plan of vertical differentiation is followed, teachers would be well advised to move slowly in this direction. Further experimentation and research is sorely needed before vertical differentiation can be safely prescribed as a solution to the problem of the rapid learner.

EDITOR'S NOTE. Professor McMeen gives a very frank discussion of the problems and difficulties encountered when one seeks to reach a more ideal arrangement of pupils and materials so that each may progress at a reasonable rate. It is not an easy task. "Grouping" is very useful. We need to explore a number of aspects of the larger problem, e.g., (a) the levels of content, (b) the modes of teaching and learning, and (c) modes of evaluation. It is unfair to all types of pupils to expect the same total program, the same levels of achievement, and the same modes of learning. Mr. McMeen has indicated a direction toward which we might work. Perhaps we should more fully explore the values to be found in the "ungraded" classroom particularly in the primary grades. Also, let us not insist on holding back our talented pupils in grades four, five, and six just so that they will not be removed from their "peer group." As Mr. McMeen has pointed out, the more able pupils need not become snobs. In fact, the opposite is usually the case. Let us continue to explore ways of meeting the needs of the different levels of pupils we find in our elementary schools.

Research on Arithmetic Instruction—1958

J. Fred Weaver
Boston University School of Education

Two years ago The Arithmetic Teacher contained a summary of research on arithmetic instruction for the six-year period, 1951–1956.[1] This was extended on an annual basis one year later, covering research for 1957.[2] The present summary represents a continuation of the previous ones and brings them up-to-date through the calendar year of 1958.

This 1958 summary is organized in two major sections. The first of these is devoted to *published* research reports; the second, to unpublished *doctoral* theses and dissertations.

SECTION I: PUBLISHED RESEARCH REPORTS

The annotated references in this section include research on arithmetic instruction that was published in periodicals, monographs, and the like during the calendar year of 1958. A serious attempt was made to make the listing as complete as possible. Omissions, if they exist, are unintentional.

In preparing the 1958 listing the writer applied the same criteria of delimitation that were used in the 1951–1956 and the 1957 summaries. Thus, published references included in the bibliography are restricted to: (1) normative and experimental studies which report specific data or findings on a problem associated with, or related closely to, mathematics instruction at the elementary-school level (grades 1–6); and (2) bibliographies, summaries, and more or less critical discussions which relate in whole or major part to such normative and experimental studies. It is hoped that these criteria were applied with acceptable fidelity when deciding whether or not to include a particular study in this 1958 summary.

[1] Weaver, J. Fred. "Six Years of Research on Arithmetic Instruction: 1951–1956." The Arithmetic Teacher 4: 89–99; April 1957.

[2] Weaver, J. Fred. "Research on Arithmetic Instruction—1957." The Arithmetic Teacher 5: 109–118; April 1958.

Annotated Listing of Published Research Reports

1. Aftreth, Orville B. "The Effect of the Systematic Analysis of Errors in the Study of Fractions at the Sixth Grade Level." *Journal of Educational Research* 52: 31–34; September 1958.[3]

Reports an experimental study involving 289 pupils in seven sixth-grade classes. Over a three-month period children in the control groups received instruction in addition and subtraction of common fractions and worked 19 sets of practice exercises, each having 20 examples. During the same period children in the experimental groups received comparable instruction but their practice work took the form of identifying and correcting errors in 19 sets of exercises, each having 20 examples with accompanying answers. Experimental hypotheses were tested in terms of the criterion variable based on both an immediate recall test and a delayed recall test. Findings prompted the conclusion that exposing pupils to errors in addition and subtraction of common fractions did not affect their learning of these operations adversely.

2. Brown, Kenneth E. *Analysis of Research in the Teaching of Mathematics: 1955 and 1956.* U. S. Department of Health, Education, and Welfare; Office of Education. Bulletin 1958, No. 4. Washington, D. C.:

[3] This same study was reported earlier by the author in the April 1957 issue of The Arithmetic Teacher under the title, "Shall We Expose Our Pupils to Errors?", pp. 129–131.

U. S. Government Printing Office, 1958. 73 p.

Lists and summarizes 123 studies (54 master's theses, 49 doctor's dissertations, and 20 projects of college and university faculty members) reported in response to a questionnaire regarding research on mathematics instruction at all levels—kindergarten through college. The 40 studies relating to the elementary-school level are discussed on pp. 16–19 of the monograph, and some of their implications are considered on pp. 22–23.

3. BUSWELL, G. T. "A Comparison of Achievement in Arithmetic in England and Central California." THE ARITHMETIC TEACHER 5: 1–9; February 1958.

Reports a normative study based on test performance for a cross-section sample involving 3191 pupils from 91 schools in England and a random sample involving 3179 pupils from 70 schools in Central California. Both samples were drawn from children within the chronological age-range of 10 yr. 8 mo. to 11 yr. 7 mo. at the time of testing. The English children were tested with a more or less "standard" British arithmetic test of 100 items: 40 non-verbal computational items and 60 verbal exercises and problems. The American children were tested with a special 70-item adaptation (31 non-verbal items and 39 verbal items) of the British test, selected from among the 100 items as those suitable for American usage with a minimum of change. For comparative purposes, the English children were scored on only the 70 items used in the test adapted for American children. "To summarize the factual results of the study, it is clear that pupils at age eleven in English schools are markedly superior to pupils of the same age in California in arithmetical achievement as measured by the 70 item test. In fact their mean score on the test was more than double the California mean, 29.1 as compared with 12.1." Sub-findings are presented also, and implications of the study are discussed. As stated in the *Editor's Note* on this research, "Dr. Buswell's article should be studied carefully and critically."

4. CORLE, CLYDE G. "Thought Processes in Grade Six Problems." THE ARITHMETIC TEACHER 5: 193–203; October 1958.

Reports a study involving data collected from 74 sixth-grade children (40 in one class, 34 in another) who each read and solved eight "typical" sixth-grade-level verbal problems. Tape recordings were made of all oral aspects of the testing situation for each pupil, and were analyzed along with corresponding written work and solutions. The data were treated in relation to accuracy of performance, in relation to each of five behavioral factors, and in relation to two different school locales. Summarizes 11 findings, 14 conclusions, and six implications based on the investigation.

5. DURR, WILLIAM K. "The Use of Arithmetic Workbooks in Relation to Mental Abilities and Selected Achievement Levels." *Journal of Educational Research* 51: 561–571; April 1958.

Reports an experimental study designed to determine " . . . the value of workbooks in promoting learnings concerned with quantitative relationships, arithmetic problem solving, arithmetic vocabulary, and fundamental arithmetical operations." Data were derived from the performance of 102 pupils (50 girls and 52 boys), representing the entire population in grades 4–8 of a small-town Illinois elementary school. The 26-week experimental period " . . . was set up on an alternate methods basis so that each student was measured in arithmetic growth [by the "Analytical Scales of Attainment in Arithmetic"] both with and without workbooks." Data were presented in support of these general findings. "The students, as a whole, made greater gains in arithmetic vocabulary and fundamental operations after using workbooks. The students who were above average in intelligence and subject matter achievement profited more from using workbooks in the above two areas, while those who were below average made no significantly different gains with or without workbooks. The mean arithmetic achievement gain favored workbooks in grades four and five, but there were no significant differences in grades six through eight."

6. ERICKSON, LELAND H. "Certain Ability Factors and Their Effect on Arithmetic Achievement." THE ARITHMETIC TEACHER 5: 287–293; December 1958.

Reports a study of the relationship of factors such as intelligence, reading ability, and socio-economic status, to arithmetic achievement in general; and the relationships and inter-relationships among intelligence, arithmetic concepts, and problem solving ability. Data were derived from the performance of 269 sixth-grade children (in eight classrooms from four schools) on appropriate standardized tests, coupled with relevant supplementary information. Pertinent correlation analyses were based not only on the sample as a whole, but also on three subgroups within the sample: upper 27%, middle 46%, and lower 27%. Various sets of findings, conclusions, and implications are reported for each section of the study but are not summarized in an overall way.

7. FEDON, J. PETER. "The Role of Attitude in Learning Arithmetic." THE ARITHMETIC TEACHER 5: 304–310; December 1958.

Reports an experimental study of the attitudes of 32 third-grade children toward arithmetic in general and toward several specific aspects of arithmetic work (problem solving and computation). Attitudes were measured by the Dutton scale in conjunction with a rather unique "color intensity scheme." Differential analyses of the data were made, leading to these observations among others: "It would seem apparent that very definite attitudes are being expressed, both for and against arithmetic as early as third grade. . . . There is a general feeling that various aspects of arithmetic are enjoyable and necessary, but not always meaningfully significant [sic]." Furthermore, the children who were studied felt that parents had a generally favorable attitude toward arithmetic and that teachers had a strongly favorable attitude toward arithmetic.

8. FLOURNOY, FRANCES. "Interpreting Definite Quantitative Statements Occurring in Reading Reference Materials." *The Elementary School Journal* 58: 208–211; January, 1958.

Reports a study of the occurrence of definite and indefinite quantitative expressions in social-studies reading material, along with an account of procedures used to improve pupils' ability to interpret such expressions. Based on an examination of a random sample of 15 pages from the 1953 edition of a fifth-grade book in a social-studies series, indefinite quantitative terms were found to be used 366 times, an average of about 24 times per page. Also, eight references were found on these 15 pages to maps, tables, graphs, etc. requiring the interpretation of definite quantities. Recommendations are made on the basis of the text analysis and the procedures used with a class of children to improve the abilities under consideration.

9. GLENNON, VINCENT J. *What Does Research Say about Arithmetic?* Revised Edition. Washington, D. C.: Association for Supervision and Curriculum Development, 1958, 77 p.

This revised edition of Dr. Glennon's well-known monograph (first published in 1952 and the object of six printings) contains a bibliography of 210 items which have been brought well up-to-date. As in the earlier edition, research findings relative to major issues in arithmetic instruction are summarized and discussed in any easy-to-read and relatively non-technical style. The following are typical of the questions considered in the monograph: What do we know about the grade placement of topics? Why is it important to develop meanings in arithmetic? Is the preparation of teachers of arithmetic adequate? Does grouping facilitate learning? Is there a place for flash-cards in a modern arithmetic program? Should children memorize the multiplication tables? How do attitudes toward arithmetic grow?

10. GUNDERSON, ETHEL. "Fractions—Seven-Year-Olds Use Them." THE ARITHMETIC TEACHER 5: 233–238; November 1958.[4]

[4] This is an extension of an earlier study, "Fraction Concepts Held by Young Children," reported by the same author in the October 1957 issue of THE ARITHMETIC TEACHER, pp. 168–173.

Reports findings from a study in which 23 second-grade children were given an introductory lesson on fractional concepts and then independently worked and made up problems involving fractions. Draws conclusions relative to the development of the fraction concept among children in the primary grades.

11. HARTUNG, MAURICE L. "Selected References on Elementary-School Instruction: Arithmetic." *The Elementary School Journal* 59: 170–172; December 1958.

The current edition of this annual *Journal* annotated listing embraces references to both significant theoretical discussions and research studies relating to various phases of mathematics instruction at the elementary-school level.

12. HUNNICUTT, C. W., and WILLIAM J. IVERSON (editors). "The Third 'R'." *Research in the Three R's.* New York: Harper and Brothers, 1958, pp. 347–429.

Summarizes each of 23 studies—"a sampling of the most significant research"—under three major chapter headings: "Insight Versus Connectionism—The Role of Drill," "What and When in Arithmetic," and "Other Areas of Methodology." Summaries of individual studies range in length from 1 page to 11 pages. The Hunnicutt-Iverson volume should prove to be a decidedly helpful contribution to our body of research literature on arithmetic instruction.

13. JOHNSON, J. T. "Whither Research in Compound Subtraction?" THE ARITHMETIC TEACHER 5: 39–42; February 1958.

Reacts to an earlier discussion by Weaver regarding research on compound subtraction,[5] taking exception to some of his interpretations.

14. KENNEY, RUSSELL A., and JESSE D. STOCKTON. "An Experimental Study in Teaching Percentage." THE ARITHMETIC TEACHER 5: 294–303; December 1958.

Reports an experimental study on instruction in the topic of percentage at the seventh-grade level, involving approximately 475 children. Equated groups were formed in order to implement three instructional approaches: (1) a drill and rote learning approach, with no attention to mathematical meanings or understanding; (2) a mathematically meaningful approach, with no mechanical rules and no drill or abstract practice; and (3) a composite of approaches 1 and 2. The experimental instructional period for each group covered four weeks. Children were measured with a specially constructed test of 55 items (25 problems and 30 abstract examples), built in three comparable forms and administered just prior to the beginning of the experimental period, at the close of the experimental period, and after a lapse of six weeks during which time there was no instruction in the topic of percentage. On the basis of measured gains, there was some evidence to indicate instructional advantages for the composite approach. This evidence, however, was neither consistent nor conclusive. The researchers offers hypotheses to account for this fact, along with suggestions for further investigation.

15. KREISMER, CLIFFORD R. "Glen Rock Scores on a Canadian Test." THE ARITHMETIC TEACHER 5: 216–217; October 1958.

Reports results of administering the following test to the 219 sixth-grade children in public schools of Glen Rock, N. J.: "Arithmetic Test: Fundamental Operations" for Grades 4–8, prepared and published by the Department of Educational Research, Ontario College of Education, University of Toronto. The average test score for the Glen Rock children had a corresponding grade equivalent of 7.9, which was a full year above the Canadian norm for a similar grade level. However, this seeming advantage in favor of the Glen Rock children all but disappeared when corrections were made for the relatively high median IQ (112.8) of the Glen Rock pupils.

16. MILLER, G. H. "How Much Time for Arithmetic?" THE ARITHMETIC TEACHER 5: 256–259; November 1958.

[5] Weaver, J. Fred. "Whither Research on Compound Subtraction?" THE ARITHMETIC TEACHER 3: 17–20; February 1956.

Reports findings from a survey of the daily time allotment for the study of arithmetic in grades 1 to 6, based on returns from 34 large-city school systems and 44 small-city school systems (over half of which were in Illinois) in response to a questionnaire " . . . sent to school systems all over the country." Found less time allotted, and greater disparity in time allotted, in grades 1–3 than in grades 4–6. Recommended that more time be devoted to the study of arithmetic, particularly in grades 1–3, and that there be greater uniformity among schools in time allotment for the study of arithmetic.

17. RAPPAPORT, DAVID. "Preparation of Teachers of Arithmetic." *School Science and Mathematics* 58: 636–643; November 1958.

Summarizes and discusses some of the significant recent findings and recommendations relating to the mathematical background and preparation of elementary-school teachers, and their attitudes toward arithmetic.

18. RAPPAPORT, DAVID. "Understanding Meanings in Arithmetic." THE ARITHMETIC TEACHER 5: 96–99; March 1958.

Reports a study of the mathematical understandings possessed by children in grades 7 and 8 and the relation between understanding and computational skill, based on data gathered from administering a specially constructed "Test on Basic Concepts and Processes of Arithmetic" and Sections E–H of Test 4 of the "California Arithmetic Test," Intermediate Form, to 271 7B pupils and 110 8B pupils. Concluded, among other things, that: "The seventh and eighth grade pupils did not have an adequate understanding of the meanings in arithmetic, assuming that a score below 50 per cent was an indication of inadequacy"; also, "Computational skill was not an indication of the understanding of the meanings of the processes used in the computation." Offers suggestions and recommendations for narrowing the gap between skill and understanding.

(The doctoral dissertation on which this report is based is listed as No. 14 in Part A of Section II of this summary of "Research on Arithmetic Instruction—1958.")

19. SAX, GILBERT, and JOHN R. OTTINA. "The Arithmetic Achievement of Pupils Differing in School Experience." *California Journal of Educational Research* 9: 15–19; January 1958.

Reports a study comparing the mathematical understandings and computational skills of children from a "progressive" school in which formal arithmetic is not taught prior to grade 5 with children from a "conventional" school. Appropriate forms of a specially constructed test of mathematical meanings were administered to children grades 3–6 of a "progressive" school, to children in grades 3–6 of a "conventional" school, to children in grades 7–8 who had come from a "progressive" school, and to children in grades 7–8 who had come from a "conventional" school. Children from "progressive" schools were matched with those from "conventional" schools on relevant experimental factors, resulting in the formation of 21 matched pairs from grades 3–4, 22 from grades 5–6, and 54 from grades 7–8. The mean difference in scores on the test of mathematical meanings favored the "progressive" children at each of the three grade-levels but was statistically significant only at the grades 7–8 level. Differences in computational skill between "progressive"-school and "conventional"-school children, as measured by the appropriate form and section of the "California Achievement Test," were studied on the basis of 18 matched pairs from grades 3–4, 19 from grade 7. At the grades 3–4 level the "conventional"-school children were significantly superior in computational skill; at the grades 5–6 level the difference still favored the "conventional"-school children but was not statistically significant; at the grade-7 level the difference favored the "progressive"-school children but not to a statistically significant extent. The conclusions drawn by the researchers are admittedly tentative. (In this connection the writer wishes to call attention to the rather small sample that was

used in the study, and to the fact that problem solving as an aspect of arithmetic ability was not measured. He also wishes to say that much more needs to be known about the validity of the test of "meanings" and the characteristics and instructional programs of the "progressive" and "conventional" schools before the reported data and findings can be interpreted with any degree of confidence.)

20. STOKES, C. NEWTON. "80,000 Children's Reactions to Meanings in Arithmetic." THE ARITHMETIC TEACHER 5: 281–286; December 1958.

Reports on a two-phase curricular development and evaluation study. In the first phase, 566 teachers collaborated with the researcher in developing a mathematics curriculum for grades 1–8 that was in keeping with a meaningful approach to instruction, with the scope and sequence of content determined on a social-utility basis. This work required a 10-year period of time, 1940–1950. The second phase of the study—a five-year period, 1950–1955—was devoted to implementation of the newly developed " 'cone of experience' method in meanings." During one year of this period, 2000 children at each grade level, 1–6, who were taught in accord with the "cone of experience" plan were matched with children taught in accord with a conventional curricular plan. Appropriate forms of the "Metropolitan Arithmetic Test" were administered at the beginning and end of the school year. On the average, children following the "cone of experience" plan scored about one year above those following the conventional curricular plan at each of the grade levels, 1–6. Also, special measures of sustained attention favored children in the experimental "cone of experience" group. The researcher concluded: "Now . . . it appears that we have evidence that a program of meanings will improve learning in arithmetic."

21. ULRICH, LOUIS E., SR. "Division by a Two-Figure Divisor." THE ARITHMETIC TEACHER 5: 204–210; October 1958.

Reports on methods and procedures developed for teaching and learning to divide by a 2-digit divisor, with test results from two classes (5A and 5B) in which this instructional plan prevailed.

22. WAGGONER, WILBUR. "Improving the Mathematical Competency of Teachers in Training." THE ARITHMETIC TEACHER 5: 84–96; March 1958.

Describes a three-semester-hour college-level course in mathematics designed especially for elementary-education majors, with data regarding the performance of students (132) who were measured with the "Davis Test of Functional Competence in Mathematics" both at the beginning and the end of the course. The mean increase in number of items correct (7.7) was significantly different from zero statistically. Also, the mean score on the test increased from the 50th percentile (based on end-of-year norms for grade 12) at the beginning of the course to the 71st percentile at the end of the course (based on the same norms).

23. WEAVER, J. FRED. "Research on Arithmetic Instruction—1957." THE ARITHMETIC TEACHER 5: 109–118; April 1958.

This is the counterpart of the current summary, covering the 1957 calendar year.

Sources of Research Reports

Within a short period of time THE ARITHMETIC TEACHER has become the source of the majority of current research reports published relative to mathematics instruction at the elementary-school level. Of the 23 reports just summarized for 1958, 14 appeared in THE ARITHMETIC TEACHER.

The Journal of Educational Research and *The Elementary School Journal* each carried two arithmetic research reports during 1958. The remaining five research reports appeared in as many different sources.

Types of Research Reports

Seven of the 1958 research reports were clearly research summaries or discussions in one form or another. The remaining 16 reports involved some form of either experimental or normative research, with half the investigations being of each type.

Areas of Research Emphasis

Among the research reports published during 1958, what aspects or phases of arithmetic received greatest attention or emphasis? The answer to this question can be inferred from the following tabulation. Although some references might have been classified in several ways, each published research report has been placed in just one category in the table below. The numerals at the right are not frequencies, but refer to specific research reports in the basic listing.

Arithmetic Achievement 3, 6, 15
Attitudes toward Arithmetic 7
Common Fraction Concepts 10
Compound Subtraction 13
Deferred Instruction in Arithmetic 19
Division with 2-Digit Divisors 21
Exposure to Error 1
Mathematical Meanings, Meaningful Instruction 18, 20
Percentage 14
Problem Solving 4
Summaries of Research 2, 9, 11, 12, 23
Teacher Education 17, 22
Time Allotments for Arithmetic Instruction 16
Use of Workbooks in Arithmetic Instruction 5
Occurrence of Quantitative Statements in Social Studies 8

Sustained Research Effort

To what extent have persons who contributed to the published research literature during 1950–1957 continued to contribute research reports during 1958? Of the persons whose names appeared in the writer's two previous summaries, 11 made some published contribution to our research literature on arithmetic during 1958. Less than half of these 11 persons reported any new experimental or normative research, and only one person reported an investigation of interest on a wide scale.

SECTION II: UNPUBLISHED DOCTORAL THESES AND DISSERTATIONS

Graduate students' theses and dissertations represent one of our important sources of research on arithmetic instruction, particularly at the doctoral level. Unfortunately, however, much of this work never appears in published form. It thus escapes the attention of all too many persons. It is hoped that this section of the current summary may help in some way to remedy this situation.

For several reasons this section is organized a bit differently than was the case in the 1957 summary, and is presented in two parts:

Part A is an annotated bibliography of doctoral dissertations on arithmetic that were listed and summarized in the 1958 issues of *Dissertation Abstracts*.

Part B is a listing of titles of dissertations on arithmetic that reportedly were completed for degrees awarded during 1958, but which have not yet, or will not, appear in *Dissertation Abstracts*. This information was gathered from returns to a questionnaire sent out by the writer to about 50 graduate-level institutions.

Although this form of organization for Section II is not the same as that used in the 1957 summary and will cause some duplication of listing, it is felt that the present form of organization will be a more satisfactory one to use, both now and in the future.

Part A. The following 16 doctoral dissertations appeared in the indicated issues of *Dissertation Abstracts* during the 1958 calendar year.

1. BEAN, JOHN ELLIS. "The Arithmetic Understandings of Elementary School Teachers." *Dissertation Abstracts* 19: 708; October 1958. (Ed. D., Stanford University, 1958. L. C. Card No. Mic 58–3758.)

Used Glennon's "Test of Basic Mathematical Understandings" as the criterion measure applied to 450 Utah elementary-school teachers. Found the mean rights-score to be about 66% of the total items in the test. Found a positive relationship between teacher self-perception of arithmetic understanding, both before and after testing, and actual test scores; self-perception was somewhat more realistic after testing than before, however. Found a small cumulative increase in teachers' arithmetic understanding with greater experience. Found intermediate-grade teachers scoring higher than primary-grade teachers in regard to understandings of common and decimal fractions

and the rationale of computation. Found a seemingly causal relationship between a teacher's arithmetic competence and her enjoyment in teaching arithmetic.

2. BJONERUD, CORWIN E. "A Study of the Arithmetic Concepts Possessed by the Preschool Child at the Time of Entrance into Kindergarten." *Dissertation Abstracts* 18: 1314–1315; April 1958. (Ed. D., Wayne State University, 1957. L. C. Card No. Mic 58–1172.)

Used as criterion measures two specially constructed tests, one oral and one written, administered to 100 children (60 boys and 40 girls) within the first 2½ months of their kindergarten school year. Summarized 13 findings and conclusions, including one that pointed to a slight advantage held by boys in comparison with girls in relation to number concepts possessed upon entrance into kindergarten.

3. BORSUCH, BARBARA JANE NEARY. "Sibling Resemblance in Reading and Arithmetic." *Dissertation Abstracts* 19: 1288–1289; December 1958. (Ph.D., University of Michigan, 1958. L. C. Card No. Mic 58–7686.)

Data derived from four analyses (three longitudinal and one cross-sectional) of test scores and related information on 294 children from 120 families. In general, concluded that " . . . for rate of achievement growth in reading and in arithmetic, siblings and families are significantly more alike than unrelated individuals and different families."

4. BURRY, HAROLD ELLSWORTH. "An Analysis of Early American Arithmetic Textbooks Through 1810." *Dissertation Abstracts* 18: 2049–2050; June 1958. (Ph.D., University of Pittsburgh, 1958. L. C. Card No. Mic 58–2020.)

Findings are based on an historical-documentary study of 65 arithmetic textbooks published through 1810. Among other things, lists 26 separate topics commonly appearing in the textbooks studied.

5. CARTER, CLEO DORIS. "The Relationship between Personality and Academic Achievement (Reading and Arithmetic) of Seven-Year-Olds." *Dissertation Abstracts* 19: 1027; November 1958. (Ed. D., Indiana University, 1958. L. C. Card No. Mic 58-5204.)

Findings and conclusions based on data from 196 seven-year-olds who were given the following tests: California Test of Personality, California Short-Form Test of Mental Maturity, and California Achievement Test; socio-economic status was determined by using the Warner Status of Characteristics Index. "It must . . . be concluded that there is no significant relationship between personality (personal and social adjustment) and academic achievement (arithmetic and reading), when the latter measure is adjusted for difference in intelligence." It also was concluded that " . . . the correlation of academic achievement with intelligence and personality jointly does not appear to be significantly better than the correlation of academic achievement and intelligence alone."

6. EVANS, FORREST FURMAN. "The Effects of a Summer Camp Arithmetic Enrichment Program." *Dissertation Abstracts* 18: 163–164; January 1958. (Ed.D., George Peabody College, 1957. Publication No. 24,466.)

Findings and conclusions based on data from 30 matched pairs of boys, one group of whom had six weeks of summer camp arithmetic enrichment experiences. Found that boys with the enrichment experience lost significantly less in arithmetic ability during the summer than did those without the enrichment experience, with less loss in arithmetic reasoning than in arithmetic computation. Found that the greater the participation in enrichment experiences, the less the loss in arithmetic ability. Found no significant difference in amount of participation in enrichment experiences between high and low IQ groups. Found that boys in high IQ group lost more in arithmetic ability than those in low IQ group.

7. HUDSON, FLOREINE HERRON. "A Study of an Enrichment Program in Arithmetic for Children in the Fourth Grade." *Dissertation Abstracts* 18: 958–959; March 1958. (Ed.D., Alabama Polytechnic Institute, 1957. Publication No. 23,967.)

Findings and conclusions based on data from 610 fourth-grade children, separated into experimental and control groups, over a 12-week period. Concluded that: "The typical class should show gain in arithmetical reasoning ability when enrichment is employed. The class may be expected to advance six or seven months within a three-month period of instruction. The normal children should profit more than the others. Although all intelligence levels did not show gains, no group may be expected to suffer any hardship from the use of enrichment materials." Also concluded that: "Teachers find enrichment materials useful for stimulating and maintaining interest. Children seem to gain more desirable attitudes toward arithmetic through using enrichment materials."

8. JONES, EMILY KATHRYN. "An Historical Survey of the Developmental Treatment of Vulgar Fractions in American Arithmetics from 1719 to 1839." *Dissertation Abstracts* 18: 138–139; January 1958. (Ph.D., University of Pittsburgh, 1957. Publication No. 24,745.)

This historical-documentary study involved 110 books which were analyzed in six chronological periods: the period prior to 1789, and each of the five succeeding decades to 1839. Specific findings obviously cannot be mentioned in this annotation.

9. KNIGHT, LYMAN COLEMAN. "A Study of the Effectiveness of the Subject Matter of Modern Mathematics in the Preparation of Elementary School Teachers." *Dissertation Abstracts* 19: 1299; December 1958. (Ed.D., University of Pittsburgh, 1958. L. C. Card No. Mic 58–5616.)

Findings and conclusions based on data involving 59 college students taught by the same professor—30 of whom were "... taught a course in elementary mathematics with the content and spirit of modern mathematics," and 29 of whom were "... taught a traditional college review course of the fundamentals of arithmetic, basic ideas of geometry, and algebra." It was concluded that: "For prospective elementary teachers with little mathematics background in high school, the subject matter of modern mathematics is more effective than traditional mathematics in developing mathematics reasoning and in developing computational skills." Also: "The preference for mathematics, and the attitude of the students in class is more favorable with the subject matter and point of view of modern mathematics than with that of traditional mathematics."

10. LOWRY, WILLIAM CARL. "The Implications of the Theory of Operationism and of Some Studies in Psychology and Anthropology for the Teaching of Arithmetic." *Dissertation Abstracts* 18: 961; March 1958. (Ph.D., Ohio State University, 1957. L. C. Card No. Mic. 58–549.)

Findings and conclusions based on analysis of relevant literature and research, and a study of the way in which 13 present-day arithmetic series develop early number experiences. Recommends greater emphasis upon an operational development of mathematical meanings in relation to the manipulation of objects, and greater attention to the recognition of small-group sizes by visual perception ("subitizing") in place of counting.

11. MCDERMOTT, LEON ANSON. "A Study of Some Factors that Cause Fear and Dislike of Mathematics." *Dissertation Abstracts* 19: 71; July 1958. (Ed.D., Michigan State University, 1956. L. C. Card No. Mic 58-2362.)

Findings and conclusions based on a case-study method applied to 41 college students, "34 of whom had developed a fear of mathematics and seven of whom were proficient in the subject." Among the numerous findings and conclusions, special attention is directed to this one: "Most students having fear and dislike of mathematics met with frustration in the elementary grades."

12. O'DONNELL, JOHN ROBERT. "Levels of Arithmetical Achievement, Attitudes toward Arithmetic, and Problem Solving Behavior Shown by Prospective Elementary Teachers." *Dissertation Abstracts* 19: 1300; December 1958. (Ed.D., Pennsylvania State University, 1958. L. C. Card No. Mic 58-7296.)

Findings and conclusions based on data from 109 college seniors in elementary education who were measured by the California Achievement Test, Mathematics Section; the Remmers "Scale to Measure Attitude toward Any School Subject"; and an oral problem-solving interview procedure. Cites 10 major findings and conclusions, these among them: " . . . attitude toward arithmetic and a liking for or a disliking for the subject appear to be somewhat different concepts. Students sometimes score high on an attitude toward arithmetic scale but expressed a dislike for the subject. . . . Attitude toward arithmetic showed only a low correlation with arithmetical achievement and arithmetical problem solving behavior."

13. POST, RICHARD. "A Study of Certain Factors Affecting the Understanding of Verbal Problems in Arithmetic." *Dissertation Abstracts* 19: 90–91; July 1958. (Ph.D., Columbia University, 1958. L. C. Card No. Mic 58-2242.)

Findings and conclusions based on a study of the problem solving behavior of 176 fifth- and sixth-grade children in relation to six factors: size of numbers, superfluous numerical data, familiarity of setting, number of steps, type of operation, and symbolic terms. "The results of the study indicated that the three most important factors affecting the understanding of verbal problems were the type of operations involved in the problem, the familiarity of the problem setting, and the presence of superfluous numerical data in the problem. The study did not yield any conclusive results on the effects of the remaining three factors although there was some evidence to indicate that the effect of symbolic terms was negligible."

14. RAPPAPORT, DAVID. "An Investigation of the Degree of Understanding of Meanings in Arithmetic of Pupils in Selected Elementary Schools." *Dissertation Abstracts* 18: 1322–1323; April 1958. (Ed.D., Northwestern University, 1957. Publication No. 24,938.)

See reference No. 18 in Section I of the present summary of "Research on Arithmetic Instruction—1958" for an annotation of this study.

15. RUDD, LONIE EDGAR. "Growth of Elementary-School Teachers in Arithmetical Understandings Through In-Service Procedures." *Dissertation Abstracts* 18: 947; March 1958. (Ph.D., Ohio State University, 1957. L. C. Card No. Mic 58-565.)

Findings and conclusions based on data from 30 teachers who participated in an eight-session in-service course built around the development of 72 arithmetical understandings. The effectiveness of the in-service course was measured in various ways, with "strong emphasis . . . placed upon the improved classroom procedures of these teachers as a measure of their growth in arithmetical understandings." Offers conclusions and recommendations regarding programs of in-service education relative to arithmetical understandings.

16. SHIPP, DONALD EUGENE. "An Experimental Study of Achievement in Arithmetic and the Time Allotted to Development of Meanings and Individual Pupil Practice." *Dissertation Abstracts* 19: 492; September 1958. (Ph.D., Louisiana State University, 1958. L. C. Card No. Mic 58-2850.)

Findings and conclusions based on data from four matched groups at the fourth-, fifth-, and sixth-grade levels that followed four different developmental-practice time-proportion patterns during a 12-week period. These four patterns allotted the following proportions of class time to developmental work, with the remainder of class time in each pattern devoted to practice work: 75%, 60%, 40%, and 25%. Pre- and end-experimental measures of arithmetic ability were made with the "Silver Burdett Achievement Tests," which provided scores on arithmetic understanding, on computational skill, on problem solving, and on overall arithmetic achievement. "Significant differences were found in the achievement of the sections in all three grades. . . . Pupils in the groups that devoted 75 per cent or 60 per cent of their class time to developmental work achieved significantly higher on total

score than pupils in groups that devoted a lesser per cent of class time to developmental work. Pupils in these same groups achieved significantly higher on understanding arithmetic and on computational skill. Differences between the treatments on problem solving were not found to be significant to this study. Tests of the significance of the interaction effects of the treatments and levels [of mental ability: high, average, low] did not show the effects of the treatments to be different at the three ability levels. The results of this study seem to justify a conclusion that if 60 to 75 per cent of arithmetic class time is devoted to developmental activities in the middle elementary grades then pupils will tend to show maximum achievement on a general achievement test in arithmetic."

Observations Regarding Section II, Part A

It was observed that almost one-third of these 16 studies dealing with or related closely to mathematics instruction at the elementary-school level were completed at two institutions: three at the University of Pittsburgh and two at The Ohio State University. The remaining 11 studies were completed at as many different colleges and universities.

As seen in the tabulation below, the area of teacher education was investigated more frequently than any other area. Four of the 16 studies were devoted to one or more aspects of the mathematical background, preparation, and competence of elementary-school teachers at either the pre-service or the in-service level.

Arithmetic Achievement in Relation to Personality 5
Attitudes toward Mathematics 11
Enrichment Programs and Experiences 6, 7
Mathematical Understandings Possessed by Children 14
Number Abilities of Young Children 2, 10
Problem Solving 13
Sibling Resemblance in Arithmetic 3
Teacher Education 1, 9, 12, 15
Textbook Analyses from an Historical Standpoint 4, 8
Time Allotments for Developmental and Practice Work 16

Some of these 16 studies appear to have more than mean import for the teaching and learning of arithmetic, and are deserving of careful scrutiny and possible further investigation.

Part B. The studies listed below represent those, in addition to any that appeared in *Dissertation Abstracts* to date, that were reported to the writer as having been completed in the field of arithmetic in connection with doctoral degrees awarded during the calendar year of 1958.

1. CARLOS, CARMEN B. "A Study of Some Basic Guiding Principles in Teaching Selected Aspects of Elementary Arithmetic with Implications for Educational Practice and Teacher Education in the Philippines." (Ed.D., Indiana University, 1958.)

2. CHASE, CLINTON I. "The Relationship of Certain Skills and Intellectural Factors to Problem-Solving in Arithmetic." (Ph.D., University of California at Berkeley, 1958.)

3. DAMGAARD, GENEVIEVE TURNER. "A Study to Determine the Significance of a Mental Arithmetic Program for Grade Six." (Ed.D., Colorado State College, 1958.)

4. DENNY, ROBERT MAY. "A Two-Year Study of the Effects of an Increased Time Allotment upon Achievement in Arithmetic in the Intermediate Grades." (Ed.D., Colorado State College, 1958.)

5. LAWSON, JOHN HERBERT. "The Construction and Revision of an Arithmetic Vocabulary Test for Grades Four, Five, and Six." (Ed.D., Boston University, 1958.)

6. LENO, RICHARD S. "Children's Methods of Problem Solving in Arithmetic." (Ed.D., Stanford University, 1958.)

7. SPENCE, EUGENE S. "Intra-class Grouping of Pupils for Instruction in Arithmetic in the Intermediate Grades of the Elementary School." (Ed.D., University of Pittsburgh, 1958.)

From all indications at least six of these seven investigations will be listed and summarized in one or another of the 1959 issues of *Dissertation Abstracts.*

Concluding Statement

The following statements were made by the writer in connection with his two previous research summaries for THE ARITHMETIC TEACHER:

"Numerous things are needed to improve the effectiveness and impact of our future research efforts. Among these only two will be mentioned. . . . First, there is need for a more thoughtful identification of the truly crucial issues or problems we face in the teaching and learning of arithmetic in the elementary school. Second, there is need for more coordination of research attack from various sources upon these commonly recognized crucial aspects of arithmetic instruction."[6]

"Those who engage in research work should sense a definite obligation to do at least two things: (1) to report their research so that it may be of wider benefit than would be the case otherwise, and (2) to report their research in such a way that it actually tells the research story adequately. The overall quality of arithmetic instruction will be improved in direct proportion to the extent that research workers accept these obligations in good faith."[7]

It was encouraging to the writer to find these points of view reinforced by Brown during the past year, when he stated:

"Research reflects three important needs in mathematics education. *First, the identification of the crucial problems. . . . Second, greater coordination of effort in attacking the identified problems. . . . Third, publication and wide distribution of research.*"

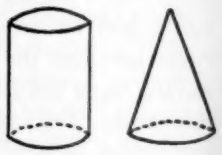
"Future advancement in the teaching of mathematics will depend upon the extent to which we identify the crucial problems, coordinate our efforts to solve them, and make the results known to the classroom teacher."[8]

[6] Weaver, J. Fred. "Six Years of Research on Arithmetic Instruction: 1951–1956." THE ARITHMETIC TEACHER 4: 99, April 1957.

[7] Weaver, J. Fred. "Research on Arithmetic Instruction—1957." THE ARITHMETIC TEACHER 5:118; April 1958.

Volume of a Cone in X-Ray

Manipulative materials: A cone made of cellulose or some transparent material that will retain its shape, a cylinder made of the same material having, of course, the same height and circumference and sawdust to be used to fill the figures.



Having learned how to find the volume of a cylinder, pupils can through the following performance proceed to an understanding of how to find the volume of a cone. Let the pupils fill the cone to the brim with sawdust and then pour it into the cylinder. Upon measuring they'll lend accuracy to their observation that only $\frac{1}{3}$ of the cylinder is filled, and through further experience pupils will see that the cylinder holds three times as much as the cone. Incidentally, it will help considerally toward clarification if the pupils mark each third of the cylinder with a color. Through questioning as well as actual performance, pupils will arrive at the conclusion that since the cone is $\frac{1}{3}$ the size of the cylinder, it will logically follow that the volume of a cone will be $\frac{1}{3}$ the volume of a cylinder, and hence the formula: $V = \frac{1}{3}\pi r^2 h$.

The advantage to be found in the use of these manipulative materials is the direct experience the pupils have in seeing as well as participating in actual solution of their problem.

Contributed by
Sister M. Vincent
Cleveland, Ohio

[8] Brown Kenneth E. *Analysis of Research in the Teaching of Mathematics: 1955 and 1956.* U. S. Department of Health, Education, and Welfare; Office of Education. Bulletin 1958, No. 4. Washington, D. C.: U. S. Government Printing Office, 1958. p. 23.

Providing Mental Arithmetic Experiences

FRANCES FLOURNOY*
University of Texas, Austin

BOB, A FOURTH GRADE STUDENT, is moving along with the cafeteria line. He is trying to add up his bill as he goes along. He wants to keep within the 35¢ he has to spend for lunch. His soup costs 7¢, sandwich 15¢, milk 8¢. A few more steps and he will be at the dessert counter. Ice cream is 8¢. He doesn't want to hold up the line. What should he do? He is trying to figure, "Is there enough money left for ice cream?"

The teacher was reading to the third grade class about the kangaroo. Jane heard the teacher say, "The kangaroo is about seven feet tall and can bound along in leaps of 20 feet each." Jane knew that seven feet was taller than her dad, but she wondered about 20 feet. Is that a very long leap? Then she heard the teacher ask, "How does a length of 20 feet compare with the length of our classroom?" Jane did not know; she wondered how long the classroom was.

Paper and pencil should seldom be necessary for solving or interpreting many such simple quantitative situations occurring in activities of everyday life. Is arithmetic being so taught in our schools that pupils are made competent in handling mental arithmetic[1] situations met in other subject areas and in life outside the school? It is doubtful that even the very best teaching of arithmetic with emphasis *only* on *written* arithmetic equips pupils to handle the *mental* arithmetic situations of life.

The swing toward little or no emphasis on mental arithmetic about the beginning of the twentieth century seems to have arisen because of an over-emphasis on mental arithmetic during the latter part of the nineteenth century. In 1918, David Eugene Smith[2] wrote:

> About the middle of the last century mental arithmetic underwent a great revival largely through the influence of Pestolozzi in Europe and Warren Colburn in this country, in each case as a protest against the intellectual sluggishness, lack of reasoning and slowness of operation of the old written arithmetic. For a long time the oral form was emphasized, in America doubtless unduly so; and this was naturally followed by such a reaction that it lost practically all of its standing.

As a result mental arithmetic has received very little attention in our schools during the first half of the twentieth century. Within the last few years more attention has been given to the need for developing mental arithmetic ability; however, we do not yet seem to be providing pupils with sufficient mental arithmetic experiences to equip them to handle life's needs for this ability.

Uses of Mental Arithmetic in Daily Life

A widely accepted principle of curriculum construction is the social utility theory. This theory states that the curriculum should include experiences and content that are similar to, and which prepare the individual for, meeting the problems encountered in everyday activities. What uses for mental arithmetic are encountered in everyday life? It is a matter of common experience and observation that life presents many uses for mental arithmetic. To my knowledge, however, there has been no really thorough study of the uses of mental arithmetic in daily life. A recent study[3] reported on the uses of both

* This paper was presented at the National Council of Teachers of Mathematics in New York City, December 30, 1958.

[1] As used here *mental* arithmetic has reference to the interpreting or solving of quantitative situations without the aid of paper and pencil.

[2] David Eugene Smith, "Mental Arithmetic." *A Cyclopedia of Education*, Vol. IV, Macmillan Company, New York, 1918, pp. 195–196.

[3] Edwin Wandt and Gerald W. Brown, "Non-Occupational Uses of Mathematics." THE ARITHMETIC TEACHER, Vol. 4, October 1957, pp. 147–154.

mental and written arithmetic during a 24 hour period by 147 students enrolled in several college classes. Only applications of arithmetic in which some type of computation was involved were recorded. In this study 75% of the uses recorded were mental and 25% were with paper and pencil. The uses of arithmetic recorded were further analyzed as 48% mental with exact answers compared to 27% paper-and-pencil with exact answers and 27% mental with approximate answers compared to 4% paper-and-pencil with approximate answers. Though not a long-range nor wide-spread study of written and mental arithmetic uses, this study indicated that non-occupational uses of arithmetic may, to a large degree, require mental arithmetic.

During a period of one week, four teachers (one teacher each in grades 3, 4, 5, and 6) cooperated in a study of children's uses of arithmetic in school and outside of school. One hundred twelve children reported 116 incidents of arithmetic usage during the week. Some uses were by the whole class in other areas of the school program. It is to be expected that these children overlooked many individual uses made of arithmetic during the week though daily reports were made. Nevertheless, 83 of the 116 reported uses or 71.6% were reported as being performed without paper and pencil. Of the 83 mental arithmetic uses reported, 84.3% were reported as being solved for exact answers. It is expected that even greater use would be made of mental arithmetic if children felt confident in solving number situations without the aid of paper and pencil.

A study of the uses of mental arithmetic as reported by these children revealed the following types of uses. Though the sampling of pupils was very small, it is felt that the types of uses are typical.

1. A problem situation in which one needs to arrive at an exact answer—
   a. Bought a loaf of bread for 24¢; gave the man $1.00; checked change.
   b. How many pages to be read from page 91 to 103?
   c. With 10 in each group and 4 groups, how many Brownie bows have I made?
   d. Need $.35 for school supplies; have 10¢. How much more do I need?
   e. Left Columbus, Texas at 6:20 P.M. and arrived in Port Lavaca, Texas at 8:08 P.M. How much time did the trip take?
2. A problem situation in which one needs only to arrive at an approximate answer—
   a. In the magazine contest, it takes about 10 points to get one prize. About how many prizes can I get with 34 points?
   b. I visited my mother in the Victoria hospital 8 times while she was ill. One way to Victoria is 28 miles. About how many miles did I travel altogether?
   c. My mother ran out of gas and had to have 6 gallons delivered. They charged $2.00. Was this more than 6 gallons would cost at the service station at about 33¢ a gallon?
3. Interpreting quantitative statements, as such quantities are read or heard, in terms of a familiar "reference measure" as:
   a. Our Christmas tree is 5 feet 5 inches tall. Is this taller than our teacher?
   b. I read in the geography book that Rhode Island is 1,058 sq. mi. in area; Texas has about 267,000 sq. mi. How do these two states compare in size?
4. Miscellaneous uses of number involving no computation (meaning no use of the four fundamental processes) as:
   a. Checked clock to see if it were time for Daddy to come home.
   b. Counted money in my bank to see how much I had for buying presents.
   c. How many minutes until recess?
   d. Paid admission to the movies.
   e. Took our 6-weeks' measures of weight and height.
   f. Read the thermometer on the back porch. It was 34°.

Reading and using tables, graphs, and scales, as found in the context of encyclopedias, newspapers, and factual social studies and science books may be classified as a type of activity involving mental arithmetic.

This use for numbers was not reported during the one week study by any one of the four teachers. It is highly possible that this use of numbers was overlooked by the teachers for it is most unusual not to have some use of tables, graphs, or scales occur in the social studies of grades 4, 5, and 6 during any one week.

Though some reports were made of using mental arithmetic in interpreting quantitative statements, very few incidents of this kind were reported by the four classes. Again it is highly probable that this use of or need for mental arithmetic is frequently over-

looked. Teachers sometimes fail to take advantage of the opportunity to encourage and teach children to interpret the quantitative statements that occur in social studies and science reading materials. Ernest Horn[4] reported that in one sampling of geography textbooks, the incidence of quantitative statements ran as high as one word in seven to a page. Horn included indefinite quantities (such as, several, many, few, heavy, thick) as well as definite quantities.

Another investigator[5] surveyed the definite quantities (not including tables, graphs, and scales) occurring in a sixth grade social studies textbook and a sixth grade science textbook (as these were used during a period of four months) and in four issues of the elementary edition of *World News Week*. Twenty-six definite quantitative statements referring to heights were found. There were 79 statements involving populations. Thirty-seven definite quantitative statements about area were discovered and 47 statements were found which involved definite quantities of distance. Therefore, it was necessary to interpret 189 definite quantitative statements. The incidence of quantitative statements seems to run very high in social studies and science materials and it seems highly probably that such statements are not understood at all by many pupils; in fact, the learner may hardly be conscious of their presence.

Children's Ability in Mental Arithmetic

Does the usual arithmetic program, with its stress on written arithmetic, equip the pupil to perform simple computations and solve simple problem situations without the aid of paper and pencil? There is very little research evidence which compares ability in performing written arithmetic with performance in mental arithmetic.

In 1913[6] David Eugene Smith wrote:

> It is a fair inference from statistical investigations that a person may be rapid and accurate in written work but slow and uncertain in oral solutions. Therefore, it will not do, from a practical standpoint, to drill children only in written arithmetic if we expect them to be reasonably ready in purely mental work.

In a study[7] conducted with approximately 550 pupils in 19 sixth grade classrooms, the mean score on a 24-item *mental* computation test involving only addition, subtraction, multiplication, and division of whole numbers was 8.84. On a similar *written* computation test the mean score was 15.20. Though the same kinds of items were included on both tests, the numbers involved were smaller and the computation much less complicated on the without-paper-and-pencil mental test; yet there was more than six items of difference in the mean scores on the mental and written test. The lower mean score on the mental computation test was taken as at least partial evidence that these sixth grade pupils lacked sufficient ability to perform simple computation when working without paper and pencil.

After a period of about two months during which about ten minutes per day were given to mental arithmetic exercises, the same 24-*mental* computation test was administered again to the 550 sixth grade pupils and a mean score of 13.85 correct was achieved. This was a mean score increase of 5 items and the difference was significant at the .1% level of confidence. The increase in mean score on the *written* computation test was 2.5 items, from 15.20 to 17.62. The data seem to indicate that pupils will not achieve in mental arithmetic in accordance with their

[4] Ernest Horn, "Arithmetic in the Elementary School Curriculum." *The Teaching of Arithmetic*, p. 10. Fiftieth Yearbook of the National Society for the Study of Education, Part 2, Chicago: Distributed by the University of Chicago Press, 1951.

[5] Merle T. Ward, *Developing Reference Measures for Interpreting Definite Quantitative Statements in Social Studies and Science Materials in a Sixth Grade Class*. Unpublished Master's Thesis, The University of Texas, 1958.

[6] David Eugene Smith, *The Teaching of Arithmetic*. Ginn and Company, Boston, 1913, p. 55.

[7] Frances Flournoy, *A Study of the Effectiveness of an Oral Arithmetic Program Prepared for Use at the Intermediate Grade Level*. Unpublished doctor's dissertation, State University of Iowa, 1953.

ability to achieve unless they are given specific classroom experiences with mental arithmetic. Ability in paper-and-pencil arithmetic does not guarantee comparable ability in work without paper and pencil.

Using two sixth grade classes of about equal ability in arithmetic achievement, a comparison was made of pupils' ability to interpret definite quantitative statements.[8] An 18-item test was administered to the two classes. One of the classes had received about three weeks instruction on interpreting quantitative statements; whereas, the other class had received only incidental attention to the interpretation of definite quantitative statements occurring in social studies and science reading materials.

The test consisted of two parts. Part A consisted of such items as, "The electricity for Los Angeles comes from a dam project 250 miles away," and the pupils were to interpret the statement in terms of a 250 mile distance familiar to them, such as, "This is like it would be if Austin got its electricity from a dam project at Dallas." On Part B of the test, the pupils were expected to "round" certain large numbers and to make comparisons without using paper and pencil. The pupils were given the "rounded" area of the United States (3 million square miles) and were expected to "round" other unfamiliar given areas, such as . . . the Republic of India—1,221,880 square miles . . . and compare the two areas. The pupils were also given the "rounded" population of Texas (8 million people) and were asked to "round" some less familiar population as . . . 19,856,000 people living in Burma . . . and compare the two populations.

The class of 28 sixth grade pupils who had received about three weeks instruction on interpreting quantitative statements achieved a mean score of 15 items correct on the 18-item test. The range was from 18 to 6 items correct or acceptable. The other class of 34 sixth grade pupils who had received only incidental attention to interpreting quantitative statements achieved a mean score of 9.24 items correct with a range from 14 to 1 correct or acceptable. As expected, the class having specific experience with interpreting quantities performed much better than pupils having only incidental attention. This skill has to be taught; it isn't just caught.

[8] Merle T. Ward, *Ibid.*

Textbook Provisions for Developing Mental Arithmetic Ability

Do arithmetic textbooks provide a sufficient program of experiences with mental arithmetic? Considering the fact that perhaps 75% or more of the non-occupational uses and perhaps a large element of the occupational uses of arithmetic may be mental, it seems important to consider textbook provisions for developing mental arithmetic ability.

An examination[9] of six fifth grade arithmetic textbooks published in recent years was made. Each of the books ranged in number of pages from about 300 to 350. The highest number of pages in a single book providing mental arithmetic exercises was found to be 43, with the lowest being 9 pages. Each of the six books provided some computation practice with estimating answers, some practice in estimating answers to word problems, and practice in interpreting graphs and scales. Those books providing similar types of exercises varied greatly in the amount of practice provided. For example, in providing computation practice in estimating answers, one book was found to provide nine pages with exercises of this kind. Three other books included only one page each on this type of practice. Only one textbook included exercises on interpreting quantitative statements using "reference measures." However, two recent textbook series begin in grade six to give attention to developing ability to interpret quantitative statements. One text[10] includes seven pages

[9] Frances Flournoy, "Developing Ability in Mental Arithmetic," THE ARITHMETIC TEACHER, Vol. 4, October, 1957, pp. 147–150.

[10] Jesse Osborn, Adeline Riefling, and Herbert F. Spitzer, *Exploring Arithmetic*, Grade 6, Webster Publishing Co., St. Louis, 1957, pp. 64–65, 74, 75, 169–170, 190.

with exercises of this kind; the other text[11] provides three pages of such exercises. This survey of arithmetic textbook provisions indicates a probable inadequacy as to amount and kinds of mental exercises provided.

Suggestions for Developing Mental Arithmetic Ability

In accordance with the probable needs for mental arithmetic in everyday life, with what types of mental arithmetic experiences should pupils be provided? It appears that the following types of exercises if systematically provided through the elementary grades should contribute to developing pupil ability to handle the mental arithmetic situations met in everyday life activities:

1. COMPUTATION—This includes:
   a. Learning short-cuts to adding, subtracting, multiplying, and dividing small numbers without the aid of paper and pencil.
   b. Practice in computing for exact answers without the aid of paper and pencil both when the pupil sees the numbers to be used and only when he hears the numbers
2. ROUNDING NUMBERS—This includes:
   a. Learning reasons for and advantages in using rounded numbers.
   b. Learning to judge when to use rounded numbers and when to use the exact numbers.
   c. Practice in rounding numbers.
   d. Practice in estimating answers with rounded numbers.
3. PROBLEM-SOLVING—Including:
   a. Practice in solving, without the aid of paper and pencil, problems read by the pupil from the textbook.
   b. Practice in solving problems read aloud by the teacher.
   c. Practice in constructing problems from everyday situations (that can be solved without the aid of paper and pencil).
4. INTERPRETING QUANTITATIVE STATEMENTS—Including:
   a. Realizing the importance of properly interpreting quantitative statements found in reading reference materials.
   b. Experience in selecting suitable "reference measures."
   c. Experience in using rounded numbers and making comparisons between familiar and unfamiliar measures.
5. READING AND USING DATA presented in tables, graphs, and scales.

In giving pupils experience with short-cut methods* of performing the four fundamental processes without the use of paper and pencil, pupil sharing of different ways of thinking seems to encourage children to try short-cut or varied ways of thinking and to find ways most successful for each individual. A study[12] was made which compared the "before instruction" methods used by pupils in grade six in performing each of the four fundamental processes, without the use of paper and pencil, with the "after instruction" methods. More than 150 pupils wrote explanations of ways of thinking before and after instruction on varied ways of thinking. Before instruction which suggested ways of thinking without paper and pencil and giving pupils opportunity to share ways of thinking, the way most often used was a paper and pencil procedure. For example, in adding 43 and 28 most of the pupils were thinking without paper and pencil just as with paper and pencil. . . . "I added 3 and 8. This is 11. I put down 1 and carried 1. This made 1+4+2 which is 7. I got 71 for an answer." Before instruction in short-cut or varied methods for mental arithmetic, approximately 85% of the pupils were using a paper and pencil way of thinking on each of the four fundamental

[11] Guy T. Buswell, William A. Brownell, and Irene Sauble, *Arithmetic We Need*, Grade 6, Ginn and Co., Dallas, 1955, pp. 73–75.

* As used here a "short-cut method" means a method which differs from the regular paper and pencil method. Sometimes it may not actually be shorter but it may be an easier way of thinking when not using paper and pencil.

[12] Frances Flournoy, "*A Study of the Effectiveness of an Oral Arithmetic Program.*" Unpublished doctor's dissertation.

processes. After instruction in short-cut methods of thinking there was a wide-spread change to the use of other ways of thinking when not using paper and pencil. On the processes of adding, subtracting, and multiplying about 23% of the pupils continued to use after two months the longer paper and pencil ways as compared to the approximately 85% who had used the longer ways in the beginning. However, there was very little change to the shorter or different ways of thinking when dividing whole numbers. It is highly possible that children do not understand the division process well enough to understand the short-cut procedures or at least varied procedures which may be easier when not using paper and pencil.

After instruction on how to add without using paper and pencil, pupils were using 11 different ways of arriving at the answer including the longer paper and pencil procedure. In adding 34 and 48, the majority of pupils were using one of two methods:

(a) $30+40=70$; $8+4=12$; $70+12=82$

(b) $30+48=78$; $78+4=82$.

After instruction in how to subtract without paper and pencil, pupils were using 10 different methods. In subtracting 24 from 62, the majority of pupils were using one method of thinking which was:

$$62-20=42; \qquad 42-4=38.$$

After instruction on how to multiply without paper and pencil, pupils were using about 5 different ways of arriving at an answer. For the example, 16×11, the majority of pupils were using one of two methods. Almost half of the 150 pupils used this method:

$16\times10=160$; $16\times1=16$; $160+16=176$.

And about half the pupils multiplied 11 by one-half the multiplier; then doubled the answer, as

$$8\times11=88; \qquad 2\times88=176.$$

After instruction on how to divide without using paper and pencil, pupils generally used two ways of arriving at an answer to a division example. While the majority of pupils used the regular pencil and paper procedure for dividing 174 by 3, a few pupils (about 10%) were using the following procedure:

$$150\div3=50 \qquad 24\div3=8; \qquad 50+8=58.$$

There is no specific evidence which offers proof that pupils perform mental arithmetic more accurately if they use short-cut or varied methods than if they use the regular paper-and-pencil way of thinking. However, in the study reported there was a wide-spread change to the short-cut methods or away from the paper and pencil method with the exception of examples involving division; yet the gain in mean score on the mental arithmetic test was highly significant. This suggests that the change to short-cut or at least different methods of thinking than paper and pencil methods may have enabled pupils to give more accurate responses to mental computation.

Children may misinterpret, or fail to interpret, definite quantitative statements for one or more reasons. Often their experiences have not given them familiarity with a similar quantity which can be used as a reference in interpreting a quantity involved in a specific statement. Some children may not know how to make the comparison once they do have a familiar reference in mind. Often children are lacking in mental curiosity regarding the quantity involved or simply do not have the habit of taking the time and effort necessary to interpret quantities as these are met in social studies and science reading materials, in newspapers, and heard on radio and TV.

Attention to the interpretation of quantitative statements as the need arises throughout the schoolday is undoubtedly helpful in developing a consciousness of quantitative statements and a greater curiosity about the meaning of various quantities in certain situations. However, such occasional attention is not judged sufficient to develop skill

in interpreting quantitative statements. Time must be set aside in which attention can be centered on learning this skill. Since arithmetic textbooks seem to give pupils very little experience with learning to interpret quantitative statements, it is recommended that a set of reference measures peculiar to a certain locality should be worked out by each class under the guidance of the teacher and that pupils be taught how to use the reference measures to clarify quantitative statements. Probably the best professional reference regarding development of this skill is the *Teaching of Arithmetic*[13] by Herbert Spitzer.

The following is an example of a set of reference measures[14] developed by one sixth grade class. (Each reference was selected because the pupils were somewhat familiar with it):

400 people	—students in Bryker Woods School
200,000 people	—people in Austin
8,000,000 people	—people in Texas
170,000,000 people	—people in the U. S.
267,000 sq. mi.	—area of Texas
3,000,000 sq. mi.	—area of U. S.
1 mile	—walk the class took
5 miles	—distance to O. Henry School and back
75 miles	—distance from Austin to San Antonio
200 miles	—distance from Austin to Dallas
7 feet	—height of door in classroom
32 feet	—height of our flagpole

Recommendations

The following recommendations are offered regarding the inclusion of mental arithmetic experiences in the total arithmetic program:

1. A variety of mental arithmetic experiences should be provided throughout the elementary grades. At present, the development of pupil ability in mental arithmetic is generally receiving insufficient attention.
2. It seems highly plausible that pupils at the intermediate grade level are capable of becoming adept in handling the mental arithmetic situations of everyday life activities. However, they are likely to perform much below their level of ability in these situations unless schools provide them with definite mental arithmetic experiences.
3. Children will adopt short-cut or varied methods for thinking without paper and pencil if given time and encouragement for exploring different ways of thinking. However, they are likely to attempt to solve mental arithmetic situations by a slower, difficult pencil and paper way of thinking if the school doesn't provide experiences with other methods. Let's encourage varied ways of thinking without paper and pencil.
4. A special type of mental arithmetic needing more attention is that of interpreting quantitative statements read and heard in daily experiences in and out of school. Children need to be made more conscious of and curious about the meaning of such statements and need special attention to developing skill in interpreting these statements.
5. Present arithmetic textbooks need considerable improvement regarding the amount and types of mental arithmetic exercises included. Until textbooks include more exercises of this kind, teachers must take the initiative in doing so.

EDITOR'S NOTE. The editor heartily commends the use of more "mental arithmetic" of the type where pupils learn to derive conclusions and find answers without dependence upon paper and pencil. It is a sad tribute to our schools of a decade ago to see so many adults almost totally lost in a quantitative situation unless paper and pencil are at hand. Frequently a mathematical situation does not call for a precise answer and often an intelligent choice is all that is required. Dr. Flournoy points out that the majority of uses of mathematics seem to be of a type that should not require paper and pencil and that we ought to be doing more instruction to meet this need. She also calls to our attention that the typical paper-and-pencil procedures are not those that an intelligent person would use in many of the "mental-arithmetic" situations. Yes, children tend to learn what the schools expect of them. We can improve thinking with quantitative items and with simple computations if we work toward that end. It is most gratifying to see a group of pupils thinking, working, and discovering various modes of finding solutions and thus participating in the real process of education. We must remember too that work in arithmetic should not be restricted to one or the other of written or mental. Many circumstances call for parts of both procedures. But in all work we should like to see a great deal of thinking based upon a good understanding of mathematical ideas, concepts, and principles. To gain these ends pupils must be given an opportunity to learn and to practice.

[13] Herbert F. Spitzer, *The Teaching of Arithmetic*, Houghton Mifflin Publishing Co., 1954.
[14] Merle T. Ward, *op. cit.*

Testing for Meanings in Arithmetic

DAVID RAPPAPORT
Chicago Teachers College

GREAT PROGRESS HAS BEEN MADE during the last twenty years in the development of an arithmetic program for the elementary school that emphasizes the understanding of the meanings in arithmetic. This program looks for insightful learning based on discovery rather than mechanical manipulative skill based on drill. Unfortunately, this advance in the teaching and learning of arithmetic has not been accompanied by an adequate evaluation instrument. How does one measure understanding? Can this be done with a paper and pencil type test? Many objections have been raised against this type of test as an instrument of measuring the understanding of meanings in arithmetic. Brownell stated the most comprehensive objection to such tests when he wrote the following:

> Valuable as tests are, they are subject to certain limitations: (a) Tests are essentially artificial: they are convenient substitutes for the ultimately valid trial by functional use. This is but another way of saying that the true measure of learning is the ability to use what has been learned in practical life situations. (b) Tests yield scores which are susceptible to errors of interpretation. We may infer, for example, that the score is a measure of the total ability tested, whereas it is but a description of a particular performance in a particular situation. Obviously, the more comprehensive our testing and the nearer our testing situation approximates functional use, the fewer will be the errors of interpretation. (c) Tests may be used too exclusively. We may rely on tests to furnish indirectly evidence of growth in areas to which tests are not very sensitive. It is exceedingly difficult to prepare tests (other than essay tests) to measure certain arithmetic outcomes. If tests alone are used, these other outcomes will almost certainly be excluded from evaluation. (d) In the attempt to secure reliable measures from tests, we may pay too high a price for objectivity in scoring. Consideration of such a factor as correct principle, or evident, understanding, introduces variation in judgment and so makes for unreliable measures. If we hold to the superior value of reliable measures, we may lose much that we need in order to understand children's work habits and thought processes. If we hold to the superior value of these evidences of learning we tend to lose objectivity and reliability.[1]

Brownell's objections are valid indeed. But one must bear in mind two reservations: 1) tests should not be the single instrument of evaluation, and 2) the tests that Brownell described are standardized tests of arithmetic skills. A new type of test should be devised that meets Brownell's criticisms. It is true, of course, that a good test for the measurement of understanding of meanings has not yet been devised, although attempts in this direction have already been made and these will be discussed later. It is also true that a good test will not be devised unless many people strive to do that and continue to learn from the experience of others. If it is assumed that paper and pencil tests can never be any good, such tests will not be made.

Need for a Good Test

The lack of a good test has already had its effect upon teachers and administrators who are questioning the value of a meaningful arithmetic program because they are unable to measure its results. Brownell has been aware of this situation, for he wrote:

> More than one school system has embarked upon a program of so-called meaningful arithmetic, only to discover that on standardized tests of computation and "problem solving" pupils do none too well. In such schools officials and teachers are likely to believe that they have made a bad bargain; and school patrons are likely to support them vehemently in this belief.[2]

[1] William A. Brownell, "The Evaluation of Learning in Arithmetic," *Arithmetic in General Education*, Sixteenth Yearbook, The National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1941, pp. 235–236.

[2] William A. Brownell, "Meaning and Skill—Maintaining the Balance," THE ARITHMETIC TEACHER, 3: 129, October 1956.

This is not to say that it is not necessary for children to learn the arithmetic skills. On the contrary, Brownell's whole article was devoted to the idea that children should learn the skills as well as the meanings. It is just that the standardized tests measure skills whereas no test has been devised that measures understandings. How does one answer the critics of "meaningful arithmetic"? Brownell voices the opinion that such people should be convinced that since there are no adequate tests, they should not rely upon tests. He wrote as follows:

> We may try to convince all concerned that the instruments used to evaluate learning are inappropriate, or at least imperfect and incomplete. True, standardized tests rarely if ever provide means to assess understanding of arithmetic ideas and procedures. Hence, the program of meaningful instruction, even if well managed, has no chance to reveal directly and explicitly its contribution to this aspect of learning.[3]

Brownell does not imply that there are no means of measuring understanding. Rather than paper and pencil tests Brownell advocates such methods of evaluation as insightful observation of pupils at work, pupils' oral reports, questioning of students, and observing them as they make errors.[4]

It is this writer's firm belief that a good test to measure understanding is necessary and that it is possible to construct one. It is also his firm belief that such a test should be used, not in place of but, together with other evaluative techniques. It will require many attempts by many individuals learning from each other's experiences before a good objective test will be devised.

Meanings Tests

Several attempts at the construction of objective tests of understandings have already been made. Johnson and Trimble have given a few examples of exercises that test meanings and understandings as guides to teachers who would like to construct such tests.[5] A few tests have been made by doctoral candidates. There may be many more tests but they are not readily available. Until such tests are made available little progress will be made in improving this kind of evaluative instrument.

A pioneering attempt to construct a test to measure children's knowledge of basic arithmetical meanings was made by Glennon in 1948.[6] Feinstein's test was concerned only with the understanding of fractions.[7] This writer's test also attempted to measure the understanding of the meanings in arithmetic.[8] This test is composed of three parts. Part I has 48 exercises that attempt to measure sixteen concepts and processes. Part II has 17 exercises that measure the same meanings as Part I but are a little more difficult. Part III has only 7 exercises and these are considered to be the most difficult. Some examples from this test are hereby presented. Objections can be, and have been, made against some of the exercises. A number of them are involved and, in the opinion of some judges, too involved for children in the elementary school. But this writer holds the belief that improvements can be made if many people try to devise such tests.

[3] *Ibid.*, p. 129.
[4] *Ibid.*, p. 133.

[5] Donovan Johnson and H. C. Trimble, "Evaluation of Mathematical Meanings and Understandings," *Emerging Practices in Mathematics Education*, Twenty-Second Yearbook, The National Council of Teachers of Mathematics. Washington, D. C.: The National Council of Teachers of Mathematics, 1954. pp. 343-355.

[6] Vincent J. Glennon, "A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels." Unpublished doctor's dissertation, Harvard University, Cambridge, Massachusetts, 1948.

[7] Irwin K. Feinstein, "An Analytic Study of the Understandings in Common Fractions Possessed by a Selected Group of Sixth and Seventh Grade Pupils." Unpublished doctor's dissertation, Northwestern University, Evanston, Illinois, 1952.

[8] David Rappaport, "An Investigation of the Degree of Understanding of Meanings in Arithmetic of Pupils in Selected Elementary Schools." Unpublished doctor's dissertation, Northwestern University, Evanston, Illinois, 1957.

Difficulties to Be Overcome

The person who attempts to devise a test for the measurement of understanding of meanings in arithmetic must overcome a number of difficulties that stand in the way of a successful test. The most important obstacle is language. A paper and pencil test must use words which are themselves difficult concepts. A test for children must use words that children will understand. This necessitates an understanding of how children think. What appears to be simple to adults may be difficult for children. The test maker is never sure of the children's interpretation of the words used. How does one measure the result? Does a wrong answer indicate a lack of knowledge, or understanding, of the arithmetic concepts and meanings or of the language used? Much more needs to be understood of child psychology and of the psychology of thinking before the language difficulties in arithmetic tests can be overcome. This should not, however, be an excuse for not attempting to construct such tests. The tests will, of course, be inadequate but they will lead to a better understanding of children's thinking.

A second obstacle in the making of a good test is also based on the psychology of thinking. What evidence is there that children understand concepts in the same way as adults do even when the same words are used? The need for better understanding of psychology mentioned in the preceding paragraph is applicable here also.

The third difficulty is the interpretation of the meanings themselves. Although a number of writers will agree on the basic concepts and meanings in arithmetic that children should learn, they do not always agree on the interpretation of these meanings. This writer sent his test to a number of the leading authorities in the field of meaningful arithmetic. These writers have written textbooks on the teaching of arithmetic, textbooks on arithmetic that are widely used in the elementary schools, and they have written numerous articles on arithmetic teaching and learning for various journals. In making comments on the different exercises of the writer's test they demonstrated different interpretations of the meanings in arithmetic. Not only children but authorities in arithmetic show variations in judgments and interpretations of ideas in arithmetic. The maker of arithmetic tests must first clarify his interpretation of the meanings his test attempts to measure.

The Role of the High School Teacher

The high school teacher of mathematics must assume some of the responsibilities involved in devising an adequate test that measures the understanding of meanings in arithmetic. Many high school mathematics teachers have voiced their disappointment in the arithmetic knowledge possessed by incoming high school freshmen. They have assumed that this is the sole responsibility of the elementary school teacher. Such aloofness on the part of high school teachers will not help solve the problem. High school teachers should help elementary school teachers clarify the concepts and meanings in arithmetic that they want children to learn. This should not be left to the elementary school teachers, nor to professors of education, nor to doctoral candidates. High school teachers of mathematics have had to teach arithmetic to students in their high schools. They should also attempt to construct tests that are aimed to measure understanding rather than manipulative skill.

Summary

The arithmetic program that has come to be known as "meaningful arithmetic" has not yet developed an adequate instrument by which to evaluate the results of such a program. Despite the objections to a pencil-and-paper type test on the grounds of validity and reliability, an objective test that measures understandings in arithmetic is necessary, especially for those teachers who need assurance that their pupils are making adequate progress. Such tests have been devised mainly by doctoral candidates. These need to be improved but this can be done

after repeated attempts by interested persons. Through the combined efforts of many workers a good objective arithmetic meanings test will eventually be constructed. There will be many disappointments, of course, and most tests will not be considered completely adequate, but such efforts must be made in order to make the program of "meaningful arithmetic" really meaningful.

Sample Test Items

1. In the number 2.037 each unit of the 3 tenths is _____ times as large as each unit of the 7 one-thousandths.
2. If the number 12.76 is thought of as made up of hundredths, it would contain _____ hundredths.
4. In doing the subtraction exercise on the right it is necessary to make some substitutions of units. We must change one of the 5 thousands to _____ hundreds, then one of the hundreds to _____ tens, and then one of the tens to _____ ones. After making these substitutions we think of the minuend as containing _____ thousands, _____ hundreds, _____ tens and _____ ones.

```
 5003
-1465
```

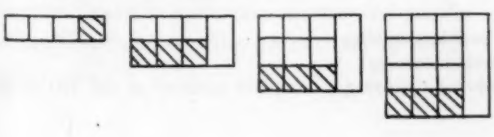
5. In doing the exercise on the right we must do the following steps:
   a) Add the _____ hundredths and the _____ hundredths. This gives _____ hundredths.
   b) Change _____ of the hundredths to _____ tenths and add this to the sum of the _____ tenths and _____ tenths. This gives _____ tenths.
   c) We then change _____ of the tenths to _____ _____. Our sum is _____.

```
  .87
+ .56
```

28. In doing the division exercise on the right we think as follows:
   a) 38 hundreds divided into groups of _____ give 6 groups. We have 2 hundreds left over.
   b) We now have 23 tens divided into groups of _____ and this gives 3 groups. We have 5 tens left over.
   c) We now have 52 ones divided into groups of _____ and this gives 8 groups. We have 4 ones remaining.
   d) Our remainder of 4 tells us that we do not have enough ones to make another group of _____.

```
  638
6/3832
 3600
  232
  180
   52
   48
    4
```

30. In adding $\frac{4}{12}$ and $\frac{4}{12}$ we add only the 4 and the 5 because they tell us how many _____ we have.
32. Draw a ring around the rectangle whose shaded portions show $\frac{1}{3}$ of $\frac{3}{4}$.



34. In order to add $\frac{4}{12}$ and $\frac{3}{4}$ we must change the fractions so that they are expressed in the same fractional units. This means that we have to change the _____ to _____.
45. To carry out this subtraction exercise we must first substitute for one of our _____ ones _____ tenths. The 3.2 can thus be thought of as _____ ones and _____ tenths. Our difference is _____.

```
 3.2
-1.8
```

47. In doing this multiplication exercise we think 6 times 4 tenths equals _____ tenths. We then substitute for _____ tenths _____ ones. We think 6 times 5 ones equals _____. We then add the _____ ones. The product is _____.

```
 5.4
×6
```

EDITOR'S NOTE. Dr. Rappaport argues well that we need good tests of understanding. These tests should measure much more than an understanding of the way in which numbers are used and manipulated in arithmetical processes. They should measure the recognition and understanding of mathematical principles and of the way they are used. This calls for a broader concept of measurement than is represented in the sample items. The forty-fifth yearbook of The National Society for the Study of Education (1946), Part 1, was devoted to the measurement of understanding and carries a chapter which lists the types of understanding that one might expect in a modern program in arithmetic. Over a twenty year period a number of people have been seeking means of measuring the understanding of arithmetic learnings by use of a printed test. Many of the items have been presented pictorially with a minimum of words. Dr. Wrightstone of New York City Schools prepared a test for primary grade pupils which aimed to measure important concepts, principles, and knowledge. As Dr. Rappaport says, it is an important field in which more work should be done.

Correction

In the February issue on page 15 the item MEYER, JEROME, FUN WITH MATHEMATICS was listed as published by World. This is correct but the "World" means World Publishing Company whose address is 2231 West 110th Street, Cleveland 2, Ohio. On page 16 the name and address were in error. This is not an item of The World Book Company of Yonkers, New York.

Efficiency in Teaching Basic Facts

EARL CLENDENON
Herzl Elementary School, Chicago, Ill.

Group pupils for work on specific difficulties

Teachers of more than thirty pupils either sigh or bristle at the much repeated advice: *plan classwork to fit the needs of each child.* For in their case it is futile to take that advice literally. But it is futile also totally to ignore that advice. An efficient compromise is to group the pupils according to three *general* levels of achievement, a technique that many teachers use successfully for reading lessons but seem hesitant to try for arithmetic lessons.

In teaching the basic number facts the following standards, if they are not too rigidly applied, are useful in grouping pupils for work on specific difficulties. Using any test on any complete set of facts (for addition or subtraction, multiplication or division) but a test in which only the answers need to be written, the results charted below are a dependable guide.

Although the pupils have been grouped as suggested, we are not ready yet to start work on specific difficulties. Pause for a while to relax in the thought that GROUP I will need only an occasional drill or test to maintain their achievement. Since one to five errors on any of these tests come from momentary slips in accuracy, if the pupil can correct such errors instantly, we ought to spare ourselves the vexation of demanding a perfect score.

But GROUPS II and III will need close attention. If we cannot give *close* attention to both groups every day, then GROUP II should receive attention first. For the wisest aim is to move as many pupils as possible, as quickly as possible, to the level of GROUP I. Proceed with these groups in the following manner.

First, steep your own mind in the realization that combinations* with *zero* or *one* need not be drilled on, and then resolve to draw the minds of the children into this realization too. Pupils are delighted when the idea of using zero as a sign for doing nothing (for *not* adding, *not* subtracting, *not* multiplying, *not* dividing) is dramatically thrust upon their imaginations. Once this is accom-

* Some authors shun the term *zero combinations* or *zero facts* and argue that these sets require no special attention. My experience runs contrary to theirs. I use the term to conserve words.

*LEVEL	ITEMS	GRADE 3	GRADE 4	GRADE 5	GRADE 6
GROUP I Adequate mastery	Score: Maximum time:	95–100 8 min	95–100 6 min	95–100 6 min	95–100 4 min
GROUP II Nearing mastery	Score: Maximum time:	85–94 10 min	85–94 10 min	85–94 8 min	85–94 6 min
GROUP III Far from mastery	Score: Maximum time:	Below 85 15 min	Below 85 15 min	Below 85 10 min	Below 85 10 min

* *Adequate mastery:* ready to progress normally with problem-solving
Nearing mastery: ready to progress slowly with problem-solving
Far from mastery: not ready to attempt problem-solving requiring a complete mastery of the 100 or 90 basic facts.

plished, the answers for such combinations as $3+0$, $3-0$, 3×0, their reverses, and $0\div3$ no longer confuse them, except for occasional slips. Persuading the pupils to understand the rule that *one* added to any number gives the next higher number (and to understand corresponding rules in the other operations) is more of a task, but a powerful time-saving procedure none the less.

Start then by insisting on a mastery of these isolated combinations *through reasoning*, not through drill. In this way only 64 out of 100 basic facts, using addition for example, remain to be learned. This is an impressive instance of the power of generalized thinking. Show the children that portion of a set of flash cards which represents their accomplishment and excite them *through praise* to appreciate fully its importance.

Use Flash Cards and Conduct the Drills Personally

We can purchase or make many devices for drill on the basic arithmetic facts. I have tried several of them, but the simplest one seems to be the most efficient one too. This is an ordinary set of flash cards. Invariably, the interest sparked by any sort of educational game or device rapidly dwindles unless the teacher skillfully directs this interest from the material itself to the learning it is intended to assist. Hence we always must be careful to judge such materials by the persistence of their effectiveness as learning aids, not by their initial attractiveness as games or diversions. To accomplish this transfer of interest *efficiently* we should develop in ourselves three qualities: (1) a genuine enthusiasm for what is being taught, (2) patience and finesse in demonstrating its meaning or importance, and (3) the habit of praising the pupils for their progress in learning it, no matter how slight or gradual that may be. It is the inspiration of these qualities, emanating from the teacher, that can maintain among the pupils an interest in so routine an activity as flash-card drills.

The importance of maintaining the children's interest obliges me to recommend that the teacher conduct the drills personally. A drill conducted by a capable child can be effective occasionally when the need for a change of routine is acute, but as the practice is extended the drills tend to become inefficient. For a child has neither the experience nor knowledge to respond sensitively to the complex needs of the individual pupils taking the drill. (It is just this sensitivity or insight, combined with the art of acting on it appropriately, that makes teaching a profession.)

Keep Flash-Card Drills Brief and Pleasant

Five minutes for a drill on the basic arithmetic facts is long enough. More than ten minutes certainly is too long. By arranging beforehand the flash-cards to be used, by maintaining an efficient routine in seating the pupils, and by insisting *pleasantly* on strict attention to the drill, we escape that waste of time arising from uncertain aims, confused motion, and a decline of interest in learning. I say five minutes for the *drill*. The drill of course must be preceded by instruction (or a brief review of previous instruction) on no more than a dozen *related* number facts. For example:

2	3	4		3	4	5	6	7
+2	+3	+4		+2	+2	+2	+2	+2
—	—	—		—	—	—	—	—

4	5	6
+3	+3	+3
—	—	—

The relationships are these: (1) The first three facts are "doubles." (2) The sum in no case exceeds nine. (3) The larger addend in every case appears first, that is the reverses are excluded. In selecting facts for a single lesson the relationships among them can be any close ones we choose to emphasize, with a view to assisting the pupil's power of reasoning and his memory.

For some reason I have not determined, most children master the "doubles" more readily than many of the other facts. In addition any "double" provides a point of reference for learning the fact whose sum is *one* greater or *one* less than that of the "double." And a corresponding statement may be made in the case of subtraction or multiplication or division too, since in every instance the "doubles" are more readily learned. Let me illustrate the instructional procedure:

> *Teacher*: (showing 2+2) We know this sum. What is it?
> *Pupils:* Four.
> *Teacher:* (showing two cards for comparison) Notice in this card (3+2) that the top number is *one* more than the top number in this card (2+2). This makes the sum *one* more than the sum of two plus two. How much is *three* plus two?
> *Pupils:* Five.

The instructional part of the lesson, then, must be a careful survey of the facts to be drilled on, of their similarities and dissimilarities, of their relationships. The emphasis must be on *reasoning out* the answer to a new fact by comparing it with a fact already learned. And an actual count of groups of objects (or of pictured objects) *frequently* should be made to demonstrate the truth of the reasoning. By taking this sort of pains with instruction we can prevent the absurd difficulties many children encounter merely because the teacher has not bothered to *demonstrate repeatedly* (I say demonstrate, not tell) that there can be no difference in the sum of 3+2 and that of 2+3. (The apparent difference resides solely in the arrangement of the groups represented by the addends.)

Now the drill may be started. Take the pupils through the cards once or twice without upsetting their consecutive order. Then begin to change the order slightly on each run until a random order is attained. Keep the pace lively, but of course no faster than the children can think. Briefly reinstruct the pupils who give incorrect responses. Insist on a response to every flash, right or wrong, from every pupil. If a child is much slower or much faster in responding than the rest of his group, consider placing him in a group whose level of achievement more closely corresponds to his own.

We teachers continually complain of pupils who rarely produce a satisfactory piece of work because of their habitual neglect of detail in preparing it. A satisfactory piece of instruction is no less dependent on habitual attention to detail in executing it.

Maintain a Flow of Encouragement to the Pupils

It is not an easy task to provide for the specific difficulties of each pupil. From a test on any set of basic arithmetic facts (the multiplication facts for example) we can select a dozen or *fewer* that each pupil needs to master, starting with the easier ones and those having some close relationship; for instance, the facts whose products range from 24 to 36, after the pupil has mastered those whose products are less. With training, every pupil can make small flash-cards for his personal use.* The teacher must help each pupil select *related* facts to study, from among those he has not mastered, and all pupil-made cards must be checked by the teacher to insure that none of the answers appearing on them are incorrect. To hold the children responsible for accuracy in this instance is an unwise demand. These cards should be studied regularly at home and during *short supervised* periods in the classroom.

It is imperative that the children be taught to appreciate the importance of studying *only* the facts they have not mastered. Almost every child prefers making a hundred flash-cards to making only a dozen or fewer, for the same reason that he prefers

* Children of low ability have less trouble keeping their flash-cards properly arranged for study when the answers alone have been written on the reverse sides, so that the faces retain a distinctive look. Cards 2¼×4 inches, bound with a rubber band, slip neatly into a boy's shirt pocket. These can be cut from tagboard or 5×8 inch file cards.

playing with a hundred pennies to playing with fewer than a hundred, but it is not difficult to enforce a mature attitude in this case and for the sake of efficiency it is necessary. Indeed, through reason, the children can be brought to enjoy discarding their flash-cards one by one as mastery over them is attained.

A thorough understanding of the function of *personal* flash-cards is a primary source of encouragement to the pupils as they use them, casting away those they finally master and putting new ones and harder ones in their places, until surprisingly only five or six remain to be learned.

Each pupil needs also some record of his progress in learning the facts. A wall chart of individual scores on the tests is useful provided the posting does not deprive us of time for instruction. But requiring children of low ability to keep their own records of scores is a futile demand. A nice way around this problem is to staple together each pupil's tests day by day, after each one has been scored and corrected. From time to time the children enjoy leafing through their files and boast over the rise in their scores. A few children like to make little bets without stakes, against the teacher, that their scores will improve by a certain amount on a certain day. But overuse of this technique spoils its effect.

The teacher needs a large stock of vigorous compliments. *Good! Bravo! Look at that boy dig those facts!* Lots of colorful remarks that may catch the fancy of the child particularly addressed—and touch his heart. These should never be lavished upon the children, but an advance in score of just a point or two is none the less an advance, and it deserves some praise. A bantering flow of encouraging comment during the drills, with a little jesting, is very effective too.

If we have not succeeded well in teaching the basic number facts the fault unquestionably lies in our methods, or in our attitudes toward this task, or in our devoting too little time to it. Certainly it is not an exciting task and its nature necessitates a meticulous routine, but it need not be a dull task or one inefficiently performed. And nothing can inspire us more than the charming willingness of our pupils to learn when we supply them with the best possible reason to learn: the assurance that they *are learning* and that *we care.*

Editor's Note. Mr. Clendenon gives some advices on mastery of the "basic facts" of arithmetic. He senses the need for efficiency so that pupils are not constantly referring to indirect methods in order to perform a simple computation. But note that his advice is for the drill or practice period which follows the initial discovery and learning. Teachers may wish to check their groups in relation to the time chart which he provides. It seems that he is reasonably generous in time so that all but the very slowest in normal reaction rate should be able to provide answers in the charted time. He is concerned with using a number of interest maintaining devices so that pupils will want to learn and to be proud of their accomplishment. Yes, we still need to learn to compute and the basic number combinations are building blocks upon which computation is based. Our children will learn if we honestly try to teach them and expect them to learn.

NCTM and NEA at St. Louis

The National Council of Teachers of Mathematics plans two excellent programs as parts of the National Education Association Convention in St. Louis. Both meetings are July 1, 1959, in the Melbourne Hotel. At 9:30 A.M. a panel, principally administrators, will give ideas and plans for taking care of the pupil who is superior in mathematics. This will be followed by questions and discussion from the floor. The luncheon speaker will give an overview of the status and goals of various groups now working to improve curricula and methods in mathematics.

Meanings in Multiplication

H. C. CHRISTOFFERSON
Miami University, Oxford, Ohio

AS ARITHMETIC BECOMES a subject in which understanding of meanings, rather than mere manipulative skill, becomes an important objective, it becomes important that teachers have clearly in mind the complete connotation of concepts.

Recent statements about the meaning of addition imply that it is an error to think of adding as increasing a quantity. If I add $2+3=5$, I have not increased the 2 nor the 3, I have merely put them together to make 5. Addition is thus a *regrouping*, rather than an increase. This is often true, yet sometimes addition does mean an *increase*. If I add 10 lbs. to my weight, unfortunately I increase my poundage. If twins are born, 2 is added to the family and there is an increase. If $200 is added to my salary, I get an increase. Then too, addition has a subtractive meaning sometimes which could be called "*subtractive addition*" or "additive subtraction" as you wish. If Jane has 15¢ and Mary 25¢, how much must I give Jane to make them equal? Here I add something to one number, I increase it, to make it equal to another number.

Subtraction has at least four distinct meanings, in spite of one author's feeling that there is only one, a separation into two groups which he contends does not decrease anything. Generally speaking, most authorities speak confidently of two basic meanings, "*take-away*" and "*difference*" and two other related connotations. George has 50¢ and spends 35¢, how much has he left? George weighs 90 lbs. and Henry weighs 75 lbs. What is their difference in weight? One meaning involves taking away, reducing, and the other involves comparison, no reduction. When one illustrates subtraction by having 5 blocks and moving 2 to one side leaving 3 blocks in the group, this is only one kind of subtraction. When I write out a check and subtract the amount from my balance, that balance decreases.

Then too, difference may have an *additive* meaning. Henry weighs 75 lbs., but normal weight for his age and height is 90 lbs. What is the difference or what should he gain in weight? Or, before he was sick, Henry weighed 90 lbs. How much did he lose? Here one subtracts the 75 from 90 to get 15, yet in actuality, the meaning is one of difference not one of take away 75 from 90.

Finally, there are 40 children in a room and 18 of them are girls. How many are boys? This is subtraction as *separation into component parts*. It resembles "take-away," but the girls are not really taken away, and there is no reduction in the number of children.

Division has at least four meanings. These are illustrated by the following examples.

1. Measurement:

   $\$6 \div \$2 = 3$

2. Partition:

   $\$6 \div 2 = \$3$

3. Ratio:

   $\$6 \div \$24 = \$6/\$24 = 1/4 = .25$ or 25%

4. Reverse of Multiply:

   $\$6 \div .05 = \$120$ from

   $.05 \times ? = \$6$

Note that measurement (1) and ratio (3) are alike in that the dividend and divisor are both concrete and the quotient abstract.

While the term ratio could apply to either one, the first one more simply means, "how many \$2 items can one buy for \$6." While the second one cannot mean "how many \$24 items can one buy for \$6," but "\$6 and \$24 have what ratio," or what number multiplied by \$24 equals \$6, the answer is $\frac{1}{4}$ or .25 or 25%.

Note also that the second and fourth meanings both involve an abstract divisor, and that the dividend and quotient are expressed in the same units. In one \$6 is divided into 2 parts, hence the name partition. In the other \$6 is not divided into 0.05 parts, yet there is a number which when multiplied by 0.05 does equal \$6. Therefore, \$6÷0.05 is a different concept from \$6÷2.

Attention is called to these varied meanings to emphasize the danger involved in narrowness and oversimplification of definitions. It is perfectly in order to begin with a simple concept, and then expand it. We do that constantly in mathematics and in science. For example, we define an exponent in a^2, then expand it to include a^{-2}, $a^{2.5}$, a^0, etc. Then too, it seems sensible to accept new meanings as the practical need for them arises, not to force all implications and connotations on a child at once as is done to the reader in this article. Multiplication is such a term. It has simple beginnings but leads on into needed aspects which even mathematicians fail to accept. There may be a few disagreements with the meanings here presented for addition, subtraction and division, but the discord is nothing compared with what will come with defining multiplication so that 2 ft.×3 ft.=6 sq. ft. or, since time×rate=distance, then 4 hr. ×25 mph=100 mi.

Understanding Multiplication

Multiplication in its simple setting is *shortened addition of equal quantities*. Two times \$6, or \$6 multiplied by 2, means \$6+\$6. The multiplier 2 indicates the number of equal quantities that are added and therefore it must be abstract and an integer. Furthermore, the multiplicand and product will usually be concrete and will always be in the same unit of measure. The argument that operations are done only with abstract numbers, and that appropriate units are then attached to the product, does not solve nor simplify the problem, it merely complicates it. It seems simple enough to say that \$6+\$6+\$6+\$6+\$6+\$6 can be abbreviated to six times \$6. With this basic meaning, if concrete quantities are involved at all, one factor which is called the multiplier must be abstract. 6×\$6=\$36, 6×6 sq. ft.=36 sq. ft., 6×6 acre ft.=36 acre ft., 6×6 man hours=36 man hours, 6×4 credit points =24 credit points.

This concept of multiplication as a shortened form of addition in which the multiplier is both abstract and an integer and indicates how many times the multiplicand is used as an addend, runs into difficulties as soon as fractions appear as multipliers. 2×\$6=\$6+\$6=\$12. But $\frac{1}{2}$×\$6, how can \$6 be used $\frac{1}{2}$ times as an addend? Here $\frac{1}{2}$×\$6 must be associated with $\frac{1}{2}$ of \$6 and that with \$6÷2.

Even before this contact with fractions the alert 3rd or 4th grade teacher encounters 32 bottles of milk at 5¢ a bottle before she has taught how to use a two-digit multiplier. She would like to say that 32×5¢=5×32¢ because the children can do the latter. This is the first extension of the definition, a famous law of mathematics, the law of commutation, plus a slight variation. Since 32×5=5×32, then 32×5¢ must equal 5×32¢. Or, 32 rows with 5 cents in each row would equal 5 columns with 32¢ in each column, if 160 cents are arranged in rows and columns. Such juggling of concrete and abstract factors is good mathematics.

A third concept of multiplication involves an abstract multiplier that is not an integer and is encountered as soon as fractions and decimals appear: $\frac{1}{2}$×6 ft., .05×\$180, 86% of 250 bushels. The original definition must give way to the wheels of progress and to the demands of common usage. $\frac{1}{2}$, .05, 86% cannot be the multipliers which indicate "how many times the multiplicand is used as an addend." 6×$\frac{1}{2}$ ft. is easy. That means just 6 pieces each $\frac{1}{2}$ ft. long to make a total

length of 3 ft. But $\frac{1}{2}\times 6$ ft. fails to fit the definition. Often teachers merely say "x" means "of" and just substitute. A sort of a proof for it could be as follows.

$4\times 2/3 = 2/3+2/3+2/3+2/3 = 8/3$

$4\times 2/3 = 2/3\times 4$ By commutation

$\therefore 2/3\times 4 = 8/3$ By substitution

But 2/3 of $4 = 8/3$ Previously learned

Consequently $2/3\times \$4 = 2/3$ of $\$4 = \$2\ 2/3$

Also $2\ 2/3\times 4 = \$4+\$4+2/3$ of $\$4 = \$10\ 2/3$

But where is the multiplier that tells how many \$4 values are added? The fractional multiplier indicates the "part of" the concrete factor that is added. This seems a simple extension, and fits decimals and percents also.

A fourth basic conception which permits both factors to be concrete or even more than two factors each to be concrete is in common use. It shall be the purpose here to show the need for it and the actual use made of it in science and industry today. While analogy is not rigorous reasoning, the situation can be well represented by analogy to algebra. Whatever the letters a, b, and c may represent (and this assumption may not be allowed by all readers) the following products are accepted.

$2a\times 3a = 6a^2$

$6a^2\times 5a = 30a^3$

$2a\times 3b = 6ab$

$6ab\times 5c = 30abc$

If $r = 5a$ then $\pi r^2 = \pi(5a)^2 = 25\pi a^2$

Yet we are not permitted to use,

2 ft.$\times$3 ft.$=$6 sq. ft.

6 sq. ft.$\times$5 ft.$=$30 cu. ft.

2 ft.$\times$3 in.$=$6 ft. in. a reasonable concept though unused.

1 ft.$\times$2 in.$\times$16 ft.$=$32 sq.-ft. in. or 32 bd. ft.

2 acres$\times$3 in.$=$6 acre inches or 1/2 acre ft.

If $r = 5$ in., then $\pi r^2 = \pi(5\text{ in.})^2 = 25\pi$ sq. in.

The physicist does not hesitate to use

6 ft.$\times$5 lb.$=$30 ft. lb.

5 hr.$\times$40 mph$=$5 hr.$\times$40 mi./1 hr.$=$200 miles, since time$\times$rate$=$distance

Here the term one hour cancels out, realizing that 40 mph is a ratio 40 mi. to 1 hr. or 40 mi./1 hr. It corresponds to $5a\times 40b/a = 200b$.

The factory cost accountant uses 6 men $\times$5 hr.$=$30 man-hours in computing costs of labor.

The irrigation engineer multiplies the area in acres by the depth in feet to get acre-feet of water. 100 acres$\times\frac{1}{2}$ ft.$=$50 acre ft.

The chief defense for this extended meaning of multiplication is the greater accuracy of thinking and computation that it induces. For instance when the irrigation water is computed, the depth 6 in. or 6 ft. is a bit different. Keeping the denomination with the number avoids error. For example, a board foot in lumber measure is 1 ft. by 1 ft. by 1 in., and could mathematically be called a square-foot inch, yet lumber dimensions are usually given in ft. by in. by in. A plank is described as a 2 by 12 which is 16 ft. long. The 2 and 12 are both inches. The measure is 16 ft.$\times$12/12 ft.$\times$2 in.$=$32 bd. ft.

Note the security and simplicity of this computation in comparison to the roundabout thinking in using all factors abstract except one. 16 ft. by 2" by 12"$=$? bd. ft. If 1 ft. long and 1 ft. wide and 1 in. thick the board would contain 1 bd. ft. This piece of lumber is 16 ft. long, 1 ft. wide and 2 inches thick, therefore it contains 16$\times$1$\times$2$\times$1 bd. ft. or 32 bd. ft. Contrast this with 16 ft.$\times$1 ft.$\times$2 in.$=$32 bd. ft.

We glibly say that rate times time equals distance or acceleration times time equals velocity. In this thinking both rate (or velocity) and acceleration are ratios. The formulas are $RT = D$ and $at = v$. To think

such a problem through without accepting the definition of multiplication that would allow all factors to be concrete, the mental gymnastics are reminiscent of the 18th Century.

PROBLEMS—

(1) If one travels 40 mph for 5 hr., how far does one travel?

(2) If one accelerates at 10 mph per minute, how fast is he going after 5 min.?

Solution with abstract multiplier.

(1.1) At 40 mph one would go 40 mi. in 1 hr.

(1.2) Since 5 hr. is 5 times 1 hr., then in 5 hr. one would travel 5 times as far as in 1 hr.

(1.3) Or 5×40 mi. $= 200$ mi., the distance in 5 hr.

(2.1) If one accelerates in his car at the rate of 10 mph per min. of starting from a "stand still," then after 1 min. he would be travelling at 10 mph.

(2.2) After 2 minutes he would have gained another 10 mph in speed and be travelling 20 mph.

(2.3) Since he gains 10 mph in speed during each minute then in 5 minutes he will have a speed of 5×10 mph $= 50$ mph.

Solution with all factors concrete.

(1.1) Formula $RT = D$. $R = 40$ mph, $T = 5$ hr.

(1.2) $40 \text{ mph} \times 5 \text{ hr.} = \dfrac{40 \text{ mi.}}{1 \text{ hr.}} \times 5 \text{ hr.}$ $= 200$ mi.

(2.1) Formula $at = V$, $a = 10$ mph per min., $t = 5$, min.

(2.2) $10 \text{ mph per min.} \times 5 \text{ min.} = \dfrac{10 \text{ mph}}{1 \text{ min.}} \times 5 \text{ min.} = 50 \text{ mph.}$

Note that mathematically these processes involve the following algebra.

$$\frac{40a}{b} \times 5b = 200a \text{ or } \frac{10a}{bc} \times 5b = \frac{50a}{c}$$

After the social setting for this computation is understood, the physicist does not hesitate to use concrete factors, in fact he often insists on using them because they eliminate a great source of error in thinking. For example, suppose acceleration is 10 ft. per sec.^{2} or 10 ft. per sec. per sec. and time is 2 min. then $V = 10$ ft./sec.$^2 \times 2$ min. or 10 ft./sec.$^2 \times 120$ sec. $= 1200$ ft./sec. or 1200 ft. per sec. Having the denominations there avoids the error in using 2 as though it were 2 sec., not 2 min.

Illustrations could be multiplied indefinitely which show that even though mathematicians have not yet accepted a meaning for multiplication in which all factors can be concrete, yet such a meaning is in use. Such a meaning is needed to keep pace with our atomic age. Such a meaning would greatly simplify the teaching of areas, volumes, and many problems in measurements.

There could be abuse of this process in some simple cases. Find the cost of 2 lb. of meat at 68¢ per lb. 2 lb. $\times$ 68 cents = 136 lb. cents. Of course if interpreted correctly, even this error is avoided, the price is really not 68¢ but 68¢ per lb. or 68¢/1 lb. Therefore 2 lb. $\times$ 68¢/1 lb. = $1.36. Of course it may be simpler to think, "2 lb. will cost 2 times as much as 1 lb., therefore 2 lb. will cost $2 \times 68¢ = \$1.36$." There can be abuse of automobiles too, or atomic bombs, so let's not hinder progress and accurate thinking by being inflexible mathematically and logically to social usage, to social needs, and to the extension of definitions to include equally rigorous ideas.

EDITOR'S NOTE. Professor Christofferson is associating "meaning" of a process with the "situation" or use of the process. This can result in a different set of interpretations than one would obtain from mathematical structure. Our primary aim is to produce sufficient understanding of multiplication as a process and of the "use-structure" of situations where multiplication is useful so that people will know how and when to perform the necessary operation. It is the intermixing of structure with process that creates many of our arguments. Why not, in the early learning stages, write down the denominations

(Concluded on page 166)

Differences in Arithmetic Performance

SISTER JOSEPHINA
Boston College, Chestnut Hill, Mass.

AT THE BEGINNING of each school year effective teachers appraise the knowledge retained from the previous grade. Such evaluation is highly necessary in subject areas where previous learning, as in arithmetic, grammar, and spelling, serves as the foundation for further learning. The present study attempted to measure statistically the retention of 122 fifth grade pupils in arithmetic over a span of three months or during summer vacation.

Studies of forgetting indicate loss in arithmetic in all grades. Sister M. Irmina[1] found significant amounts of loss and also errors repeated from one test to a later one. Kirby[2] suggested, as a result of his study, that during the first month of school intensive review is necessary to repair losses in arithmetic deteriorated through disuse. Finally, the emphasis on meaningful drill work was stressed by Sister Mary Adelbert[3] since conditions of ordinary life afford little opportunity for continuous repetition and, therefore, the computational skills in arithmetic suffer.

The present study is an abstract of an experiment carried out under the writer's direction by Sister Anna Eugene Leaver.[4]

Pupils from three fifth grades using the same texts and exposed to the same course requirements served as subjects. The chronological mean age was 10.9 with a sigma of .72, while the mean mental age was 10.7 with a sigma of 1.2. They were from a middle socio-economic level.

A test composed of thirty items on computation and twenty problems covering the main objectives of the arithmetic syllabus was administered in June and the same test readministered in September. There was no knowledge, by either teacher or pupils, that there was to be a retest. In fact, the September test, rearranged so as to limit the recognition of the June test, was not commented upon by any pupil as being familiar. The time allotments and testing procedures were the same in both tests administeredby the classroom teacher.

Table 1 shows the mean score in arithmetic computation and problem solving in June and in September.

TABLE 1

INITIAL AND FINAL TEST SCORES IN ARITHMETIC COMPUTATION AND PROBLEM SOLVING FOR 122 PUPILS IN GRADE 5

Measure	Arithmetic Computation		Problem Solving	
	June	September	June	September
Mean	20.91	15.09	13.39	9.36
S.D.	4.36	5.30	3.60	3.32
Range	30–1	28–1	20–3	18–1

The data in Table 1 indicate a loss in mean scores in both computation and problem solving.

Table 2 presents measures of loss along with the results of the tests of significance measured by the critical ratio.

TABLE 2

MEASURE OF LOSS AND RESULTS OF TESTS OF SIGNIFICANCE IN ARITHMETIC COMPUTATION AND PROBLEM SOLVING FOR 122 PUPILS IN GRADE 5

Measure	Arithmetic Computation	Problem Solving
Mean-Initial Test	20.91	13.39
Mean-Final Test	15.09	9.36
Loss	5.82	4.03
SE Diff.	1.09	.78
C.R.	5.54*	5.82*

* Significant beyond the .01 level.

Examining the results of the test of significance, both C.R.'s indicate a high level of confidence. The loss amounts cannot be attributed to chance factors but are in large part related to a lack of drill and use, and possibly, meaningful first learning.

Table 3 presents the per cent of failure for the items in computation.

TABLE 3

PER CENT OF FAILURE ON INITIAL AND FINAL TEST IN COMPUTATION FOR 122 PUPILS IN GRADE 5

Item Computation	Per Cent Failing Item in		
	June	September	Difference
Fundamental Operation			
Addition	32	46	14
Subtraction	14	24	10
Multiplication	17	31	14
Division			
Short	26	29	3
Long	22	48	26
Fractions			
Addition	24	43	19
Subtraction	32	45	13
Multiplication	28	42	14
Division	32	42	10
Roman Numerals	50	75	25
Mensuration	19	45	26

Table 4 shows failure data for items in problem solving.

TABLE 4

PER CENT OF FAILURE ON INITIAL AND FINAL TEST IN PROBLEM SOLVING FOR 122 PUPILS IN GRADE 5

Item-Problem	Per Cent Failing Items in		
	June Test	September Test	Difference
Fundamental Operation			
Addition	13	21	8
Subtraction	31	42	11
Multiplication	26	55	29
Division	27	46	19
Fraction			
Addition	35	44	9
Subtraction	61	77	16
Multiplication	19	29	10
Division	34	51	17
Mensuration	33	71	38

Examining Tables 3 and 4 there are progressive amounts of loss in the final test ranging from 3 per cent (division) to 38 per cent (mensuration).

In computation the greatest loss appeared for items dealing with mensuration ($\frac{3}{4}$ hr. = ? min. and 4000 lbs. = ? tons), Roman numerals (Write the Arabic number for L), and long division (1972 ÷ 29 and 38497 ÷ 48). The largest difference in per cent of error in problem solving was found in items calling for solution by use of subtraction of fractions as:

> Bill can swim across the pool in $45\frac{1}{2}$ seconds. Jack can do it in $51\frac{3}{10}$ seconds. How much longer does it take Jack than Bill to swim across the pool?

Mensuration problems utilizing perimeter, area, linear measure, and avoirdupois indicate a difference of a large percentage of error.

Summer vacation does affect retention of arithmetic as seen in the study. The results are in agreement with similar findings mentioned earlier.

In terms of the value of the critical ratios, all of which are significant beyond the .01 level of confidence, it is apparent that the amount forgotten is highly dependable. As a consequence, teachers, at the beginning of the new school year and at frequent intervals, should re-present the previously learned material for maximum retention and for future success in a progressive subject such as arithmetic. A teacher-made, diagnostic-survey type test administered at the beginning of the new school year reveals to the teacher the strengths and weaknesses of her new class. Therefore, she builds on a more pedagogically sound foundation.

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(*Concluded on page 166*)

The Theme in Arithmetic

ALAN RIEDESEL
State Teachers College, Plattsburgh, New York

THE WRITTEN THEME is another means of improving arithmetic understanding. Writing a theme affords the pupil an opportunity to express in words his ideas and understanding of an arithmetic process on which he has been working. The ability to explain the steps of a procedure is a good indication that the pupil really understands the process.

During the tenth week of school the seventh grade class was completing a review of the study of multiplication of fractions. The group had used concrete materials, various techniques for solving examples and problems, and had arrived at what they considered the best method of working with fractions.

At the beginning of one period the teacher suggested that the class could clarify its thinking concerning the multiplication of fractions by using a written description of the method that they used.

This was the first time that the students had attempted to write short papers concerned with an aspect of arithmetic. Thus, a number of them had difficulty in beginning their themes. Such statements as "I think I know a lot about the multiplication of fractions but I haven't had to really think about it until now" were common. But, after a few minutes everyone was writing and, although the writing went slowly at the beginning of the period, it was gratifying to note that once the child did begin to write, ideas seemed to come to his mind and he was able to write rather rapidly. At the end of the period the papers were given to the teacher.

The next period the papers were returned to the group and ideas concerning them were exchanged. The teacher asked the group if they would be interested in reading some of the papers written by other members of the class. The class decided that they should be rewritten in ink so that they could be easily read as a part of a display on the bulletin board. The following are examples of the compositions:

(Jay's paper)

I think that the meaning of "multiplied by a fraction" is a short way of adding fractions.

The way that you multiply a fraction by a whole number is, first you change the whole number to an improper fraction. Then you multiply the numerator and the denominator by the denominator. If necessary change to a mixed number. For example:

$$3\times\tfrac{1}{2}=$$

$$\tfrac{3}{1}\times\tfrac{1}{2}=\tfrac{3}{2}=1\tfrac{1}{2}$$

The way you multiply a mixed number by a fraction is that you change the mixed number to an improper fraction. Then multiply the numerator by the numerator then multiply the denominator by the denominator. If necessary change the product to its lowest terms. For example:

$$2\tfrac{4}{8}\times\tfrac{1}{4}=$$

$$\tfrac{18}{8}\times\tfrac{1}{4}=\tfrac{4}{8}=\tfrac{2}{3}$$

The way you multiply a fraction by a fraction. You multiply the numerator by the numerator. Then multiply the denominator by the denominator and if it is necessary change to lowest terms. For example:

$$\tfrac{3}{4}\times\tfrac{2}{8}=\tfrac{6}{32}=\tfrac{3}{16}$$

MULTIPLYING FRACTIONS

(Barbara's paper)

1. To multiply $4\times\frac{3}{4}$ you first have to change 4 into a fraction. That would make 4, $\frac{4}{1}$. Then you can cancel like this: $\frac{4}{1}\times\frac{3}{4}$. Four will go into the numerator one time. Four will go into the denominator one time. Then you multiply one times three, which is three and one times one which is one. One will go into three, three times, so the answer is three.

2. To multiply $2\frac{1}{2} \times \frac{1}{2}$ you first have to make the $2\frac{1}{2}$ an improper fraction. You do this by multiplying the whole number by the denominator of the fraction and you get $\frac{5}{2}$. Your problem now looks like this $5 \times \frac{1}{2}$. You then multiply the numerators 5 and 1, and you get 5. Then multiply the denominators 2 and 2, and you get 4. It is now an improper fraction so you divide 4 into 5 and get 1 and $\frac{1}{4}$. $1\frac{1}{4}$ is the answer.
3. To multiply $\frac{3}{4} \times \frac{5}{5}$ you first can cancel. You cancel like this: $\frac{3}{4} \times \frac{5}{5}$ the denominator three goes into itself one time and the numerator three goes into itself one time. The numerator 5 goes into itself one time and the denominator 5 goes into itself one time. You then multiply denominators 1 and 1 together and the numerators 1 and 1 together and you get $\frac{1}{1}$. Then divide 1 into 1 and you get the answer 1.

The above arithmetic themes are typical of the group's first attempt to use the theme. As the students continued their arithmetic work during the semester they were asked to write other explanations of mathematical processes. Following the completion of a unit of work on decimals the theme below was written.

WHAT ARE DECIMALS?

(*Annie Lou's paper*)

Decimals are based on ten and multiples of ten. The decimal point separates the whole number(s) from the decimal fraction.

We use decimals because it is easier to get an accurate measure with them, especially with very small measurements.

It is easier for me to use decimals, because there are not many steps to remember when I multiply, divide, add, and subtract them. It is easier for me to read them, too.

When you are adding decimals you have to remember to have the decimal points in the addends and the sum in a straight column. When you are subtracting decimals you have to remember to keep the decimal point in the minuend, subtrahend, and difference in a straight column. When you are dividing decimals you have to remember to move the decimal point in the divisor and the dividend to the right as many places as there are in the divisor. You have to remember to put the decimal point in the quotient, right, above the one in the dividend after it has been moved. When you multiply a decimal you have to remember to count how many figures there are to the right of the decimal point in the multiplicand and in the multiplier, then you count that many places off in the product.

We use decimals to show someone's batting average, to show how fast someone can swim, to show how high someone can jump, to show how long something is and to show how much something weighs.

Mr. Riedesel or who ever else reads this theme, I think you would be interested in knowing that the decimal system came into use in the seventeenth century.

Summary

In analyzing the above procedure, it can be noted that the following benefits can be derived from writing an arithmetic theme:

1. The teacher has a means of evaluating the individual student, because the themes show the variations in the level of understanding among the children.
2. A valuable tool is provided for use in diagnosing the difficulties that individual class members have with a particular arithmetical process.
3. The actual writing focuses attention on a particular skill and makes it necessary for the student to clarify his thinking on a particular arithmetic area.
4. Writing about mathematics entails the ability to develop accurate explanations and descriptions which will help the student in writing about other subjects, particularly in the scientific fields.
5. The children have a new approach to mathematics, a subject in which writing technique is not usually employed.
6. The students are helped to think about the *how* and *why* of a particular arithmetic process.
7. Class unity can be maintained at a high peak, because all of the students are able to participate in the same activity with a feeling of achievement.

While the arithmetic theme is not by any means a panacea, when properly used it can be a valuable aid in the development of a sound arithmetic program.

EDITOR'S NOTE. The written composition or theme can be very revealing in the way it reflects the level of thinking and performance of a pupil. Teachers frequently assume that pupils have developed a higher level of understanding than is really true. A tape recording or a theme, as Mr. Riedesel suggests, is helpful. The most common procedure is merely the asking of questions and listening. Perhaps we teachers do too much talking and not enough listening.

Teaching the Mentally Handicapped

FRANCIS T. FOTI
Francis Read School, Philadelphia, Pa.

DEFINING THE "MENTALLY HANDICAPPED" as those children who can be educated in special classes in public schools, Kirk and Johnson state that these children in general tend to achieve between the third and fifth grade in their arithmetic abilities and some are unable to achieve even third grade ability. Accordingly, for these pupils the logical development of number as far as the 5th grade would be in most cases:

1) Reading and writing numbers to six places, e.g. 683,495.

2) Addition and subtraction to five-place numbers

$$\begin{array}{r} 89665 \\ +\text{ or }-28723 \\ \hline \end{array}$$

3) Multiplication by hundreds,

$$\begin{array}{r} 342 \\ \times 212 \\ \hline \end{array}$$

4) Division with two-place divisors,

$$72\overline{)288}$$

A study of fractions would include under addition and subtraction:

1) The idea of mixed number $1\frac{1}{2}$, and improper fractions, $\frac{3}{2}$.

2) Adding and subtracting fractions, mixed numbers, and whole numbers,

$$\begin{array}{r} 2 = 1\frac{4}{4} \\ -\frac{1}{4} = \frac{1}{4} \\ \hline \end{array}$$

3) The concept of fractions—part of a group.

4) Multiplication of whole numbers and fractions, and fractions by a fraction,

$$\tfrac{1}{3} \times 10,\ \tfrac{1}{3} \times \tfrac{3}{4}.$$

5) U. S. money under the four processes would complete the study during the arithmetic period.

Most authorities agree that the content should be carefully chosen on the basis of two principles:

1. The content must include the knowledge, skills, and concepts that will be of more value to the pupil now and in later life, and

2. The methods used should be determined by the special disabilities or abilities of mentally handicapped children.

Regarding point one, it is the responsibility of the teacher to provide the mentally retarded child with the kind of arithmetic instruction that will help him to meet his current needs and to prepare him with those practical arithmetic skills which will aid him in earning a living, in running his household, and in interpreting his daily life experiences involving number. Arithmetic instruction, therefore, involves a very practical development so that the mentally retarded child can see sense and make use of what he learns from day to day. His daily experiences in the school, in the home, and in the community should be used to motivate what he learns. Motivation, then, is of primary importance to success in instruction.

The teacher would do well to determine what arithmetical knowledge the child already has and seize every possible social situation and opportunity from which she can provide meaningful concepts of terms relating to time, quantity, space, measurement, and commercial terms. These might evolve from an experience unit that she is developing that would bring together activities in languate arts, health, physical education, safety, art, music, and social studies, as well as numbers.

Some examples are:

1. Distances
   a. How far from home to school and return
   b. Cost of traveling daily—weekly
   c. Distance from school to a place such as Independence Hall
   d. Use of map scale
2. Class trips
   a. Distance
   b. How much bus fare does each child need?
   c. How much money does the whole class need?
   d. How long will the trip take?
   e. What time do we leave? Return?

Point two under our original two principles obviously refers to the training of a child according to his capabilities physically, mentally, socially, and emotionally, e.g., a child who stutters would have his work adapted so that he could demonstrate his answers concretely. Another child might be quite adept at sewing so that measuring might be emphasized as part of her instruction.

Costello compared the effectiveness of three methods used in teaching arithmetic to mentally handicapped children. The methods were:

1. *Socialization*—an active, experiencing method. Children's experiences are necessary to all learnings in arithmetic. Children have great difficulty in learning a fact or skill which cannot be related to their own experiences. These experiences can be drawn from the children's own past experiences, or from actual experiences in the classroom, e.g., an excursion to the zoo might consider the number of weeks, days, hours, minutes, and seconds until the trip; mileage; bus or trolley fare; weight of the animals; etc.

2. *Sensorization*—a method in which concreteness of presentation is used, e.g., a child who writes his 3's backwards might trace his finger over a piece of sandpaper cut out in the shape of that number.

3. *Verbalization*—a method in which verbal description or telling is used.

Costello found socialization to be the most effective method followed closely by sensorization. Verbalization proved to be the least effective method for the improvement of attention, association, vocabulary, comprehension, and judgment.

Cruickshank studied various arithmetical abilities of a group of mentally handicapped children and compared them with a group of normal children of the same mental age. He found the mentally handicapped child:

1. less able to differentiate unneeded from needed facts in verbal problem solving, e.g., Jack worked 5 days for $30. How much did he make a day if he worked 8 hours each day?

2. likely to indicate that he would solve a problem in one way and actually proceed to solve it in another way;

3. inferior in ability to solve exercises in the four fundamental processes;

4. with greater proficiency in addition and subtraction than in multiplication and division which would indicate that more time on practice should be spent on the basic number facts particularly multiplication and division.

5. significantly inferior in ability to define and use correct arithmetic terms. In this latest deficiency, the vocabulary peculiar to a certain process should be defined, e.g., in division, the quotient, divisor, dividend, and remainder;

6. less likely to admit inability to solve problems, resulting in more frequent attempts to proceed blindly.

Summary

The teachers of the mentally handicapped child can feel confident that an arithmetic program is desirable if it makes use of socially significant situations and includes a direct and systematic program of instruction with a minimum dependence on transfer of training for the learning of new skills.

The work should include the knowledge, skills, and concepts that will prove of value to the child now and in adulthood.

The methods used should be determined by the special disabilities or abilities of mentally handicapped children.

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EDITOR'S NOTE. It certainly is true that we should try to teach as much useful arithmetic to retarded children as we can. We need to think seriously about the nature and amount of this arithmetic. Are computations of greatest importance? Should computations be learned by the conventional methods? What elements of geometry should be learned and at what age levels? How valuable for these pupils is learning with the use of diagrams, scales like rulers, and other manipulative devices? Would it be better for these pupils to understand a comparatively few principles that are significant to people like this and to be able to think with and to use these in their lives than to try to memorize procedures that are apt to be forgotten? How can we best serve these people? As Mr. Foti points out, their travel is at a slow pace, they need crutches frequently, and they may not travel far. But it is our obligation to give them the aid that will make them as independent as possible for the role they will occupy in our society. Some twenty years ago, the editor experimented with a combination of shop work and mathematics with some retarded boys of ages ten to thirteen. One outgrowth of the experiment was the "discovery" by pupils of non-conventional methods of obtaining answers to simple computational exercises. The carpenter's steel square became a most useful device. Who has information (with data) to report interesting procedures and programs for the mentally retarded?

The Problem of Problem Solving

CLARICE SINNER
Rigler School, Portland, Ore.

IN ORDER TO BE COMPLETELY SUCCESSFUL, a good arithmetic program must stress problem solving.

To learn about problems, the pupil has to study questions as well as find answers. An effective way to study questions is to construct them and pay close attention to how they should be made. When they have found the answers to problems, children should be ready to show how they arrived at the answer and how they know the answer is correct. They then should be helped to formulate some of their own. Problems should be stated clearly. They should be concerned with, and be within the limits of the work studied, yet should be sufficiently challenging to the pupils. Children should find the facts the problem presents and then take note of the question or questions the problem asks. Pupils at first need a great deal of guidance. There are several ways in which we can help children visualize and understand a problem. This can be done by dramatization, by using substitute materials to represent articles in the problem, by discussion of vocabulary to make meanings clear, by practice in stating facts and questions for problems, and by making up problems.

Problem solving should not be separated from the other work of arithmetic, but should be a part of it at all times. The ability to solve problems that confront children every day will become easier if they have practice with actual problems that arise in the life of the home, school, and community. If problems are real, they will be of interest to children.

One of the ways in which our sixth grade class used the direct experience recently was in connection with a cafeteria problem. The

manager brought to the school's attention the fact that she was often unprepared for the number of children buying lunch because there was too much of a discrepancy between the number of children actually buying and the number that had been counted. The problem was presented to the class. They decided that although they made up only a fraction of the total school, it would be a good beginning if their count was accurate. They take their turn in the morning writing down on the board the number buying lunch, number taking, and the number going home. They add this total to the number absent. We ask other questions at this time such as: What is the largest part of the class going to do this noon? How many more children are buying than are taking their lunch today? What fractional part of the class is absent? How much money will the cafeteria collect from our room today? What part of the class is going home for lunch? What part is buying lunch? Sometimes, John buys lunch tickets for a week at a time. How much does he pay? There are many more.

We also use arithmetic in a health and science unit. We recorded each child's pulsebeat for 15 seconds and then proceeded to find the answers to questions such as these: What must we do to find the number of pulsebeats for one minute? What is considered the normal pulsebeat for boys and girls your age? Compare it with your own. What was the highest count in the class? The lowest? What is the difference between the two? Compare yours with that of a neighbor. What is the difference between your pulsebeat when you have been running and when you remain still?

We learned that about $\frac{1}{20}$ of a person's weight is blood. Each child figured how many pounds of his body consisted of blood. Comparing these with each other involves not only division, but includes much practice in fractions since the remainders must be changed to fractions. They learn here to add and subtract mixed numbers, sometimes with like denominators, sometimes with unlike denominators. Other questions involving math also presented themselves.

Much of this kind of arithmetic can and should be motivated by the teacher if it does not come readily at first from the children. The important point is that they must have practice that is well-motivated and meaningful in order to develop the ability of applying math problems in everyday life.

The direct experience is especially important in the lower grades. Number facts can best be taught from activities and experiences of the pupils. As the child goes on in school, less direct ways of learning can be used.

It is impossible, of course, always to provide direct participation in lifelike experiences for all children in the class. Consequently, it often becomes necessary to use vicarious experiences that give them this opportunity. They should be as similar to life situations as possible. Some interesting information about transportation, measurement, money, time, weather, sports, buying, selling, communication, and many other subjects may be the basis for problems.

We should remember that there is no set formula or step by step procedure that will guarantee success for all in this matter of problem solving, but these are some general reminders: Always encourage the pupil to read the problem carefully; think about what it asks and what it tells; point out to the child that he must read the problem slowly and carefully; have the child tell the problem in his own words to make sure he understands; slow him down if necessary; be patient and give the child time to think about the problem and its solution.

Editor's Note. Obviously Miss Sinner believes that children should be given experience in understanding problem situations and that one good way to do this is have them formulate problems about situations with which they are familiar. This is good advice. Understanding the mathematical relations in structure of an experience or situation develops through thinking about the essential elements of the situation. Our final aim in arithmetic instruction is to develop in people the ability to sense the mathematics in a situation, to think through the relationships, to perform the necessary computations, and finally to reach a correct conclusion. Good experience in problem solving should do much to establish this aim.

Using the "Problem-Solving" Method

PETER S. PIERRO
Pleasantdale School, La Grange, Ill.

MOST TEACHERS OF ARITHMETIC will, at one time or another, deplore the inability of their students to do word or story problems. We usually attack this difficulty by asking our students to read the problem again, read it more carefully and to find what the problem asks.

In doing these things we recognize that our problem lies not in the area of the arithmetical figuring but in the area of the reading and organization of the information. Accepting this, we may ask, "What is the best way to read our information?" It is no secret among teachers of reading that the best reading is accomplished when the individual is reading for information concerning a problem that he has in mind.

Analyze your procedure when you set forth to write a paper. The first thing you do is select or have given to you a problem. You then gather information in terms of that problem and refrain from gathering extraneous information.

This is a typical problem:

Mr. Rogers made an automobile trip. When he left home his odometer read 17,895 miles. When he returned, three days later, it read 18,325 miles. He used 25 gallons of gas. How many miles does his car travel per gallon of gas?

Unless you are rather unusual, you read the problem (perhaps scanned it) and after reading the question, you returned to reread the information. You did this because the first time you read the information, it was rather meaningless. Only when you reread the information in terms of the problem posed did it become meaningful.

The "three days" mentioned in the problem became meaningless and extraneous information only when we knew the problem.

Let's rewrite the same problem posing the problem first:

Mr. Rogers wants to find how many miles he travels per gallon of gas on an automobile trip. When he left home his odometer read 17,895 and when he returned three days later it read 18,325. He used 25 gallons of gas.

After the first sentence in which the problem is posed, the remainder of the information is read *in terms of the problem.* Mr. Rogers in having kept a record of the mileage and the amount of gasoline used had the problem in mind. It would be very unusual if he had kept this information and then, as an afterthought, decided he would like to know his gas mileage.

Here are two more problems in different settings. Read them and analyze your thinking.

No. 1

(a) **Our school yard is 200′ long and 180′ wide. What is its area?**
(b) **We want to know the area of our schoolyard. It is 200′ long and 180′ wide.**

Why was the schoolyard measured? We gathered this information because we wanted to find the area. The problem could have been: (1) What is the perimeter? or (2) How much longer is it than wide? The information gains meaning only when it is thought of in terms of the problem involved.

No. 2

(a) **My grades in math for 5 tests taken during the last grading period were 90, 84, 79, 94, and 100. What was my average?**
(b) **What was my average on 5 tests taken in math during the last grading period? My grades were 90, 84, 78, 94, and 100.**

Why don't we take advantage of this golden opportunity to start our students at a very young age to solve problems by the scientific method? Teach them first to define the problem, then to gather and organize their information (always in terms of the problem), and then to set up hypotheses, select the correct procedure and solve the problem.

Developing the "Take-Away" Idea

EVA JEAN MAPLES
Center School, Freeport, Ill.

IN MY SECOND GRADE the children reach a point when it is necessary to do problems that involve subtraction. They must use ideas that have been learned in their work and play, both at home and at school. With this as a base, I try to help the children to identify the language of subtraction so they are able to subtract intelligently.

Their first work with problems involving subtraction consists only of oral work. The children, themselves, with some questioning on my part, will describe many situations requiring take-away subtraction which I intend to teach them.

To introduce the language used in take-away subtraction problems, I used the following developmental lesson:

I placed the following statement on the board. "If we have some things, we will have fewer or less things if we:" This provoked much interest. Immediately someone said, "It isn't a sentence because it doesn't end with a period or a question mark." A child read it aloud. I told them to think about taking things away so that they could finish the sentence and tell how to get fewer or less things. The situations I provided required the use of real objects. As the following situations were described and enacted, I wrote the responses on the board. The responses were underlined in the dialogue which follows:

Teacher: "Kenneth has some marbles in his desk. Can you think of something he might do to get rid of some so that he would have fewer than he now has?"
Responses: "*Give some away.*" "*Throw some away.*" "*Lose some.*"
Teacher: "Cheryl has some candy. Can you think of some way not yet written on the board for her to have fewer pieces?"
Response: "*Eat some.*"
Teacher: "I have some eggs in this carton. How could I have fewer eggs?"
Responses: "*Use some.*" "*Drop* and *break some.*"
Teacher: "David has some new puppies. What might happen so that he would have fewer puppies?"
Responses: "He could *sell some.*" "*Some* might *run away.*" "*Some* could *die.*"
Teacher: "Can you think of any other ways that there could be fewer or less of something?"
Response: "If someone had flowers, they could *pick some.*"
(Just then one of the lights in our room went out.)
Response: "That's one. *Lights* can *go out.*"
Response: "There will be fewer pheasants when *some get shot.*" "They could *fly away* too."
Response: "Planes could *crash.*"

(The original statement, together with our list, was then read. It was noted by one child that these were all ways in which we could take things away to see how many were left, and that we did not always have to cross them out to take them away.)

Teacher: "Wouldn't it be fun if we could make some stories about each of these ways? I'll make the first one, and you can hear how I do it. Cindy has 4 pennies. She gave 2 pennies to Mary Ann. How many does she have left?"

Some of the children's problems follow:

1. Richard has 2 papers in his desk. He threw one away. How many did he have left?
2. Susan had 5 jacks. She lost 2. How many did she have then?
3. Cheryl has 6 pieces of candy. If she eats 3 of them, how many will she have left?
4. Mrs. Maples has 6 eggs. If she bakes a cake and uses 6 of them, how many will she have left?
5. There are 6 lights in our room. If one goes out, how many will be lighted?

When we finished, we had described a problem-solving situation for every item on the list. We did not try to solve any of these. The next day the children asked if they could write more story problems.

After a lesson such as this the children are quite ready to attack subtraction problems, textbook or real, and derive much pleasure from being able to do "hard problems" just like older brothers and sisters. The story problems they wrote also showed value gained in other areas of our curriculum. When given the chance, I find that second graders love to write their own stories, and their ideas are limitless.

Arithmetic Can Be Fun

NORENE HARMON, 2nd Grade Teacher
Patterson Elementary School, Patterson, Calif.

TO SOME SIX- AND SEVEN-year olds, arithmetic is a dull and fearful subject, to others fun and frolic. This is how one class made it fun and frolic: A Class Store.

The teacher introduced the use of money by having the children count their lunch money and milk money each morning. The children studied a chart of real coins, and a toy store was suggested.

A small toy store was brought into the room. The store was made of plywood about three feet long and four feet high. It had a small door and large windows with three shelves. The pupils brought some of their inexpensive toys to be placed on the shelves. Pencils, balloons and suckers were placed on the top shelf. That was the penny shelf. Articles for a nickel were placed on the second shelf and the ten cent collection was on the third shelf.

To afford an opportunity to play store and to work with varying amounts of money, the children selected two or three articles and then told their money story.

Playing store activities were continued by having the children pretend to go to the store and buy two or three toys, illustrating their purchase on paper, and then telling how much money they had spent. Other exercises were employed, too. Each child was given twenty-five cents to spend. Children then illustrated their stories and told how much change they had left. Members of the class also suggested that three nickels might be used in place of a dime and nickel. A child had to demonstrate his ability to make change before he could become the storekeeper. The same steps in making change that are used by clerks in stores were used in this activity. They started with the amount the item cost, counted pennies to the nearest multiple of five then counted nickels and dimes to a quarter. They decided it was fun using the toy store, changing the articles on the shelves and being storekeeper with access to the toy cash register. Repetition of this experience was always welcome.

During this activity the children were studying about Community Helpers. A health unit was in progress and plans were being made to visit the hospital just before Easter. It was suggested that the class take a large Easter basket filled with appropriate toys for the children confined in the hospital. Where would we get the money to buy the gifts?

A candy sale at noontime was planned. Letters were written:

March 27, 1956

Dear Mother:

We want to make some money to buy presents for sick children. Will you make some candy for next Friday? We will have a candy sale.

Thank you,
Nancy

A large chart of the plan was written and the children advertised the candy sale in each room of the school:

> We will have a candy sale next Friday at noontime. The candy will cost five cents a bag. We plan to buy presents for sick children in the hospital. We hope you will help us by buying some of the candy.

Arithmetic became real again.

"What day should we sell the candy?" (Use of the calendar)

"What time will we sell the candy?" (Use of the clock)

"What expenses will we have?" (Use of money)

"How big will we cut the pieces?" (Use of ruler)

"How many shall we put into each bag?" (Measurement)

"Who will sell the candy?" (Tally by voting)

After the candy sale, the class was delighted to learn they had $23.60. The money was carefully counted, and it was observed that 100 pennies or 20 nickels, or 10 dimes, or 4 quarters or two half-dollars made one dollar. Cash received included a one dollar bill and a check for one dollar which was cashed at the bank after proper endorsement.

Next, plans were completed for the trip to the hospital. Cellophane bags used in sacking the candy, a large potato basket to be decorated as the Easter basket, crepe paper, ribbon and colorful wrapping paper had to be purchased.

From the balance of the money, each child was given 50 cents and all trouped to the local 5-10-15 cent store to buy gifts. As readiness, the children reviewed the value of money, how change is made, and the kind of item which would be suitable for gifts for children who are hospitalized. Lists were written on the blackboard: color books, magic slates, clay, books, puzzles, paper dolls, etc., and each child chose beforehand a gift to buy, and then made a selection at the store.

Returning to class, each child showed his gift, told how much it had cost, including tax, and how much change was received from the original 50 cents. One child said, "It looks as if we now have more money in change, than we had when we started."

"Let's count it and see," said another. After counting all the pennies and nickels, it was discovered that the class had but $4.08 left. The balance used to subscribe to a children's magazine and to buy a large stuffed Easter bunny. The bunny was not to be given away, but was to be left at the hospital. A daily record of the money spent was on the blackboard as follows:

Potato basket	$ .50
Bags for candy	2.07
Wrapping paper for presents	4.13
Crepe paper for Easter Basket	.47
Bunny	2.51
Magazine	3.00
Presents	10.92
Total	$23.60

With the decorated basket filled with presents, the children, teacher and local doctor visited the hospital.

Later the hospital staff wrote a thank-you letter to the class, and the children learned the big bunny (named Candy by the children) was given to the sick children to hold in their arms on the way to the operating room. Since each child wrote his name on the present he bought and wrapped, many children have received individual "thank-you" letters.

The local radio station learned of this activity, and the class was invited to tell of their experience over the radio. Each child was given the opportunity to tell something about this project. Appropriate songs were sung, along with this original song about the trip to the 5-10-15 cent store.

Melody: (School Days)

FUN AT THE STORE

(Chorus)

School Days, School Days, dear old
Northmead school days, Reading and
Writing and 'rithmetic, We've learned
to make change, it's really no trick,
We went shopping a few days ago,
Down to the dime store we wanted to go,
To buy for the hospital gifts for the sick,
And hope their recovery'll be quick.

An arithmetic project can be fun for children and at the same time filled with desirable learnings.

My Work with the Binary System

SANDY FRIESE (7th Grade)
Whittier Jr. High School, Lincoln, Neb.

IT SEEMS AS THOUGH it was only yesterday that my arithmetic teacher, Mr. Norton, gave me a special unit on the *Binary System* and asked me to look it over. The problems that decorated the cover were certainly strange; $10+10=100$, $1+1+1=11$, $11\times 11=1001$, and $100-1=11$.

We had been studying the decimal number system in arithmetic class and had learned that it was based on the number 10, had ten digits including zero, and that the idea of place value was important in determining the actual value of any particular digit. But I certainly couldn't imagine a system based on the number two and one that used only two digits—0 and 1.

I hadn't studied the unit long, before I found out much more about this interesting number system. I learned that the place value idea in the binary system wasn't difficult at all. Since it is based on the number 2, the place values from right to left go one's, two's, four's, eight's, sixteen's, thirty-two's and on up the scale according to the power of two being used. For example, the binary number 10011 means: 1 one, 1 two, 0 fours, 0 eights, and 1 sixteen. Thus the binary number 10011 is equal to the decimal number 19.

The working of binary problems in addition, subtraction, multiplication, and division was extremely interesting and the fact that I found out that various calculating machines actually used this system made it even more interesting.

Of course, I needed help from time to time on various parts of the unit, but the unit itself was mainly self-teaching. My two main difficulties came in the carrying process in addition and borrowing in subtraction. I worked especially hard on these two processes, but now that I look back at the time I was studying these two binary processes, I realize that it was then that I really learned the meaning of carrying and borrowing in problems in the decimal system.

When I had completed the unit, Mr. Norton gave me a quiz on its contents. I guess he was pretty well pleased with the results because he asked me to present the ideas I had learned to the entire class. All of my classmates seemed to enjoy my talk on the binary system and most of them could work the problems I gave to them at the end of my presentation.

Next, I presented a 20 minute talk to a 9th grade class at our school. I'll have to admit that I was really scared at first, but these algebra pupils were a very courteous audience and they listened and worked cooperatively all through my talk.

The next thing I knew, Dr. Milton Beckmann of the University of Nebraska, had asked my teacher and me to visit his methods class in mathematics. Of all things, Dr. Beckmann wanted me to present the binary system to his university pupils. The practice that I had in giving the talk to my own class and the 9th graders helped me a lot. Only one of the college pupils had worked with the binary system before and he as well as the others were extremely attentive and courteous. They all said that they had enjoyed my short presentation; I must say that I did too although I had the "butterflies" again for the first few minutes.

In short, I really enjoyed my experiences in working with the binary system. I felt that it not only gave me some new ideas about other number systems, but developed for me a greater appreciation of our own decimal system. I certainly hope others get the opportunity to study this unit.

Just yesterday Mr. Norton mentioned the Quinary and Duodecimal Systems. I wonder what they are like?

Arithmetic in Upper Elementary Grades

PAULINE FRAZIER AND MARGARET BURMAN
Los Angeles City Schools, Calif.

THE MOST EFFECTIVE LEARNING procedures emphasize meaning and understanding. A modern program in arithmetic aims to develop meanings so that the child perceives purpose and logic in what he is taught. When the learner can see the reasons for the uses or applications of the arithmetic that is being studied, he will be able to understand more clearly and more efficiently use the number relationships involved. This program makes use of children's experiences, a growing readiness for learning, a step-by-step course in arithmetic, and a continuous review of skills. It also guides the child to become increasingly aware of the place numbers have in the world in which he lives, such as in science or business, and provides him with opportunities to apply his newly-learned concepts in arithmetic to these everyday problems of community living.

Learning is a slow process. From the time a child is first introduced to an arithmetic process to the time he has mastered it, a relatively long space of time will have elapsed. Arithmetic introduced and learned at each grade level will be dependent upon previous learning, and it will also serve as a basis for future learning. Basic concepts are introduced in the primary grades and reapplied in more complex form in each succeeding grade. The child's readiness for new concepts is dependent upon this gradual and sequential building of experiences with meaning at each grade level from primary grades through upper grades.

Extending Lower Grade Learnings

The upper grade arithmetic program of the Los Angeles City Elementary Schools includes further practice and more complex problems in the four fundamental processes of addition, subtraction, multiplication and division of whole numbers. It includes common and decimal fractions and per cents. The modern program attempts to build an increased understanding of the orderly structure of the decimal number system, the laws by which it operates, and an appreciation of the simplicity and efficiency of the decimal number system as compared with other number systems such as the Roman numerals. As soon as we record numbers of 10 or larger we are dealing with a notational system which we received from the Hindus and Arabs. It is a decimal system which employs the important principle of "positional notation" and this can be contrasted with the principles used with Roman numerals.

The upper grade program also embraces the teaching of useful technical terms of arithmetic to be used in expressing relationships and ideas. It strives to develop an understanding of units of measures and skillful use of measurement to solve problems in real life situations. The teacher seeks to increase the pupil's ability to make dependable estimates and generalizations. The pupil uses practical applications of arithmetic to gain a greater understanding of the importance of arithmetic in intelligent buying and selling, and in solving personal and community problems.

Although children always learn better and remember longer if they fully understand what they are doing, forgetting is normal. For this reason teaching, drill, and reteaching is needed. Also each child is a unique individual with his own interests, abilities, and needs. This variation in achievement and rate of learning is characteristic of any classroom group. The child's attitude toward the subject is the result of a combination of many factors such as interests, rate of learning, aptitude, health, background of experiences, and environment.

Arithmetic becomes most meaningful when it is used in solving problems within the realm of the child's experiences, or for which he has a real need to arrive at a satisfactory solution. Therefore, children should be taught sound and understandable ways in which they can solve these problems. Home, as well as school, provides many opportunities for children to develop these problem-solving abilities. At home they can learn to plan personal budgets for their allowances or earnings and for their time. They can learn to save money in a bank account which will necessitate the use of deposit slips and other business forms. They may learn to cook by measurement and to figure the cost of preparing food. They might help in caring for and replanting or landscaping your home in order to obtain a practical understanding of measurement of time used, and cost of improvements and repairs, or help in a workshop with planning, measurement and construction. Happy arithmetic experiences which occur at home with parents enable the child to approach new classroom experiences in arithmetic with confidence and anticipation.

Studies and experience show that usually the child who is successful in arithmetic understands the meaning behind the printed symbol. He has made practical application of his arithmetic facts, and in doing so he has perceived logic and purpose for numbers in daily living. He has some knowledge of the wise use of money, buying and selling, taxes, time, measurement, and graphs. He has acquired some appreciation of the contributions arithmetic has made to other fields of endeavor such as science.

The arithmetic program of the Los Angeles City Elementary Schools is one built upon basic concepts in sequential steps with more complexity at each grade level. Since forgetting is to be expected to some degree it is necessary to reteach, drill, and find many practical applications of arithmetic both at school and at home.

Meanings in Multiplication

(*Continued from page 151*)

of the "situation structure" of a problem and perform the necessary computations according to formula or another recognized pattern which produces the correct answer? If a person understands the nature of the denominations and how *the number of them* may be used to produce in a "situation-structure" some other amount of a different denomination, why quarrel with labels and how they appear on paper? The operating numbers merely are tagged with a label to identify their significance in the "situation-structure" and the operations are performed with the numbers and not with the labels. Many rather intelligent people and most scientists use labels to identify the "situation-structure" as an aid to thinking about the relationships. Is Dr. Christofferson correct in his comparison of the furor over such ideas as "the multiplier is always abstract" with the former argument over "how many demons can dance on the point of a pin"?

Differences in Arithmetic

(*Continued from page 153*)

3. Sister Mary Adelbert, "Teaching Arithmetic Meaningfully for Permanent Retention," *The Catholic Educator*, XX (1950), 263.
4. Sister Anna Eugene Leaver, "A Statistical Study to Determine the Amount of Forgetting in Religion and Arithmetic by Fifth Grade Pupils During a Three Months Interval," (unpublished) Master's Thesis, Boston College, Chestnut Hill, 1950.

EDITOR'S NOTE. Sister Josephina has presented statistical evidence showing the amount of forgetting of fifth graders over the summer vacation period. The amount of forgetting seems alarmingly high. Is this in accord with current studies in other schools? What will we do, first, to obtain learning that is not so easily forgotten and, second, to rebuild this forgotten learning the following year? Is it wisest to start a year with a diagnosis of previous learnings and then to reteach before starting new work? Or should we proceed cautiously and rebuild as needed during the first half of the year? Some teachers start the year by using the book for the previous grade and refreshing and relearning certain critical things before progressing into new work. One cannot assume "Once learned, never forgotten." But we ought to be seeking a method of learning that results in less forgetting. The editor would like to see Sister Josephina's tests applied to a group of pupils who had spent at least two previous years in a learning pattern that featured visual and manipulative aids plus emphasis upon understanding and discovery. Would such a group be less prone to forgetting? We should have this information.

Book Reviews

Arithmetic in My World Series, Grades 1–6 (teacher editions, workbooks, students books), C. Newton Stokes. Boston: Allyn and Bacon, 1958.

The textbooks in *My World Series* are attractive and appealing to children. The mechanical make-up of the book is of a definite superior quality. All levels appear to be well edited and in the main the vocabulary well chosen. The logical arrangement of material makes the content well adapted to instructional procedures.

The illustrations and unit tests are very good. The design of the teacher's manual is excellent and in addition to its timely helps in teaching is also designed in a way that is economical of the teacher's time.

The FIRST BOOK of the series has been set up with the child in mind. Many of the problems are those he will run into and need to know how to solve himself. The book should not be used without the workbook to accompany it. The teacher needs the extra drill that is given here for the slow child and extra work for the rapid learner.

The vocabulary in the book may be a little difficult for the slow child, but with a divided group it should work out very nicely. The average and above average child will find very little difficulty in mastering any new words. A large number of the words are found in the basic reading series and thus will be of help later in the book.

One of the nice features in the book is the way it teaches multiplication along with addition and subtraction. It also gives illustrations showing how to arrive at the answer. The teacher's guide also supplies the teacher with numerous ideas and makes suggestions of what to do with the rapid learners in the room. These all help to make for a happy situation in the numbers class and this particular book meets desired objectives in a commendable manner.

The vocabulary in the SECOND BOOK seems particularly advanced from the beginning. The new mathematical terms are introduced too rapidly within a lesson. The presentation of these new words is not simple enough, nor is it followed through with sufficient drill.

The textbook and workbook (as a team), abound with realistic, colorful and interesting illustrations, but the challenging exercises and problem stories are included for the more mature children in a class. The average seven or eight year old child may not enjoy this book.

BOOK THREE is very good. The teaching of addition, subtraction, multiplication, and division at the same time seems to be an excellent way and one that children can comprehend better than each process being taught separately. The teaching of all four signs of operation simultaneously should cause less confusion as more emphasis could be placed on the sign and when it is to be used.

The workbook is essential when using this text to give the drill that is necessary.

The help given by the manual would make the teaching of arithmetic easier for an inexperienced teacher.

Many teachers may feel that more should be taught in the third grade such as carrying and borrowing. However, if everything is well taught in this grade, as covered by this text, the rest can be done faster in the next grade.

The FOURTH GRADE BOOK would be attractive to children in appearance, and from the standpoint of interest and children's needs. The presentation of addition and subtraction is done in a way that should enhance quantitative thinking. The emphasis on story problems is a good feature of the book.

Multiplication and division is presented in an isolated manner. It is felt that there

are advantages in presenting the basic facts of these processes as a group. There seems to be too much pictorial material for this grade level. If the readiness program has been well taken care of in the primary grades, about 90% of fourth graders do not need to count marks and dots to get the concept. It would seem that parts of the book are definitely geared to the minority or the 10%. It does not seem to offer the desired challenge to the high average and rapid learner groups.

In the text alone there would not be enough practice problems, however if the workbook was used in conjunction with the text this would be adequately taken care of.

The material used in the FIFTH GRADE BOOK lends itself particularly to an activity or laboratory type of class room. As the manual points out, the book is designed to have the class work in small groups.

Each concept is developed very slowly in the book, this is excellent for the slower pupils but the better groups might very easily become bored. The "point" of the process needs to come sooner in the explanation.

The textbook must be accompanied with the workbook if there is to be enough practice. The workbook provides a more useful activity for the slow, average and fast learner. This is offered by more challenging application, more complex abstract relations and different ways of finding the outcome.

Much of the enrichment work for the faster pupils is left to the teacher. It is suggested that the problem materials be brought in from local newspapers, etc. There are no "for fun" mathematical equations for these pupils to try their skill.

The units are up-to-date and the problems are those that would interest fifth graders. The textbook covers measurement very nicely but gives no mention of graphs.

Since this series explains each process so thoroughly, there seems to be no need for such a lengthy review and explanation of the simple processes at the beginning of the book.

The activities in the SIXTH GRADE TEXT are well within the range of interest of children of that grade. If practical geometry and percent is to be taught, more time will be needed than is now available.

The number concepts are presented in practically the same order as in other accepted texts and the sequence in which they are presented seem best for effective learning. The detailed development of the four fundamentals seems unnecessary if the teaching in the preceding grades has been thorough. It seems that there is too much explaining in some instances.

The interesting illustrations and exercises will make the book attractive to children. Excellent drill exercises and unit tests are found frequently throughout the book. The manual and workbook are excellent.

As a set, the series start out with an excellent first grade book, however there seems to be a gap in bridging to some of the difficult second grade material. In other books of the series there appears to be a great deal of material at the beginning that is rather elementary. There was some feeling that there is insufficient materials for individual differences.

Since we subscribe to the premise of individual differences then it would seem to be a healthy sign that reviewing teachers differ somewhat in their evaluation of the series. A number of the group however indicated that they would like to have this series at their particular grade level.

LOUISA RIPPERGER, *First Grade*
EVELYN ROCKUFELER, *Second Grade*
ELVIRA LOFGREN, *Third Grade*
PHYLLIS HUGHES, *Fourth Grade*
GLENDORA SCHULDT, *Fifth Grade*
PHYLLIS KENDALL, *Sixth Grade*
WILLIAM O. MARICLE, *Chairman*

Teaching Arithmetic for Understanding (with teacher's manual and student workbook), John L. Marks, C. Richard Purdy, Lucien B. Kinney. New York: McGraw-Hill Book Company, Inc., 1958. Cloth, xiv +429 pp., $6.00.

Research by Grossnickle and others has revealed the lack of substantial mathematical background and good procedures for the teaching of today's arithmetic. Here is a

publication which attempts to meet the challenge by professionalizing the subject matter for pre-service and in-service teachers (grades 1–8). The authors maintain theirs is a dual purpose when they say: "So that the teacher may thoroughly comprehend the mathematical aspects of elementary school arithmetic, mathematical ideas such as properties of our number system, why numbers are placed in certain positions for computation, and the principles and relations underlying the operations are stressed." and, "Above all else the emphasis here is upon strategy and techniques for use in helping pupils develop understanding of arithmetic." It is the opinion of this reviewer that the authors, on the whole, have done remarkably well in dealing simultaneously with mathematical substance and methods. Indeed, for the professionals who know Buckingham's *Elementary Arithmetic: Its Meaning and Practice* and the many articles written by Brownell and Van Engen relative to the philosophy and psychology of arithmetic, this publication may appear to be a good integration of all three.

The scope of the text is adequate. Its fourteen chapters include: 1. Why Study Arithmetic; 2. Planning Effective Learning Activities; 3. The Arithmetic Curriculum; 4. Number and Number Systems; 5. Beginning Number Experiences; 6. Addition and Subtraction; 7. Multiplication and Division; 8. Common Fractions; 9. Decimal Fractions; 10. Learning the Language of Per Cent; 11. Measures and Measuring; 12. Learning to Solve Word Problems; 13. Appraising Progress of Pupils; 14. Adjusting to Individuals. Also included is a section on Games for Fixing Skills and one on Selected References. There is a "name index" as well as a "subject index." Within each chapter the sequence and techniques for developing number ideas are commendable as are materials and devices.

Mathematically the content is weighted heavily in favor of the collectional approach to number. It would seem addition and subtraction could be strengthened and zero made more significant had the number line been given greater prominence. In our time, thinking with positive *and* negative numbers has become almost commonplace in the elementary school. Likewise more emphasis on equational thinking in items such as $n+5=12$ and $9-n=6$ as well as $\square+\square=7$ and $\triangle+\triangle=1$ would invigorate mathematical insight.

Sensible algorisms are employed in the discussion relating to common fractions and it was good to see "$\frac{1}{2}+\frac{1}{3}=?$" in the horizontal form as well as the vertical. Division of fractions is presented adequately for practical purposes but the rationale for the inverted divisor will be found in a much later section called "Experiences for Enrichment." The rationalization of multiplication and division of decimal fractions gets very extensive treatment and at times the use of the words "rule," "hypothesis," "theory," "law" and "generalization" in this section may be confusing.

Occasionally the authors lapse into the older pattern of verbalizing mathematical ideas, and sometimes it is acknowledged as such. It would seem that "borrow," "cancel," "reduce" and "to the right of the decimal point" have had their day and that in the interest of good mathematics these should give way to "change," "simplify" and "to the right of the ones place."

Following each chapter there are numerous stimulating questions and exercises succeeded by an extensive supporting bibliography. Missing, however, is reference to Brownell's "Psychological Considerations in the Learning and Teaching of Arithmetic," (Tenth Yearbook, NCTM, 1935) and Van Engen and Gibb's "Mental Functions Associated with Division," (Iowa State Teachers College, 1956.). While McConnell's "Recent Trends in the Learning Process," (Sixteenth Yearbook, NCTM, 1941) is included in the bibliography for chapter 2, none of the preceding 25 questions or exercises refer to it. Arithmeticians consider this a most significant investigation of generalization in the field of number. Indeed, it is one of the first important studies relative to drill vs. understanding and,

in the opinion of this reviewer, it deserved a focal question or exercising.

Though there is a workbook to provide further exercises as well as study questions, according to the authors, its assignments can be made for use with other arithmetic methods textbooks. The Teacher's Manual supplies answers to the text and workbook questions and it provides examples of test questions (without answers).

These materials by Marks, Purdy and Kinney make an ambitious and concentrated program intended for one semester of 45 or 30 meetings. Thus selectivity will be pertinent to insure the best use of their many suggested activities. And, while the text will assist some college instructors, it cannot be considered the only answer to a good mathematical background for elementary school teachers.

ANN C. PETERS

Understanding and Teaching Arithmetic, E. T. McSwain, Ralph J. Cooke. New York: Henry Holt and Company, 1958. Cloth, xi+420 pp., $5.50.

The purpose of this book, as stated by the authors in the preface, is to serve as a "teaching guide to teachers and prospective teachers who desire to improve their understanding of the meanings, vocabulary, and mathematical operations that constitute the language and science of arithmetic and who want . . . methods and materials . . . that may motivate and assist pupils in experiencing purpose, meaning, interests, and satisfactions from their study and use of arithmetic."

These are laudable aims. The pattern of presentation is laudable, too. So also is the aim of the authors to use each chapter "as an opportunity to think with teachers about the topics which are found in a typical curriculum in arithmetic," and to stress meanings.

Unfortunately, these aims are not achieved. The discussion is wordy, confusing and full of inaccurate statements. The level of mathematical understanding and accuracy may be indicated by a few typical excerpts:

(1) "When a person wants to record an amount he uses a number. When he desires to compare two numbers he uses subtraction." (page 6)

(2) "A dividend such as zero cannot exist because a dividend notates a sum." (page 105)

(3) "Multiplication is a thought process of finding the sum or product of a given number of like integers or fractions and notating the total with one number (the product)." (page 182)

(4) "But in statements such as 'The population increased by 6%' there is little meaning because the whole quantity, i.e. former population, is not given. Unless this is known, the population increase cannot be computed." (page 240)

In quotation (1) the first sentence is obvious, the second false. One is at least as likely to use division to compare two numbers. Concerning (2), it is perfectly possible for zero to be a dividend; it cannot be a divisor. (The reviewer has no idea of what "notates a sum" is supposed to mean.) As to (3), multiplication has nothing to do with finding sums. And finally, the quoted statement (4) has a precise and significant meaning. There are many occasions when it is quite sufficient to know rate of increase without knowing the absolute amount.

The pedagogical aspects of the book seem more meritorious: the "explanatory questions," the suggestions for teaching procedures, the questions for self-evaluation, the "suggested activities" and the exercises, all seem generally appropriate. But it is difficult to see how an adequate pedagogical superstructure can be erected on so shaky a mathematical foundation.

The strictly computational aspects of the book are substantially correct, though so many special cases must surely tend to confusion rather than to clarity of thought. The chapters on measurement, problem solving and evaluation are generally good.

The book includes also an extensive bibliography and a "basic vocabulary." The

latter includes a number of terms of little importance: e.g., abacus, are decade combination, and a number of terms quite inaccurately defined: 0 is omitted from the list of Hindu-Arabic numerals, and is not a "digit," a degree is "a unit for measuring an angle or temperature" (this gives the impression that the same unit is used for angles as for temperature), multiplication is "a mathematical and mental process of rapid addition of a given number of like numbers."

Albert E. Meder, Jr.

New York State Meeting

The New York State Council of Teachers of Mathematics will meet in Syracuse on May first and second. Among the speakers are Dr. C. Stanley Ogilvy, author of "Through the Mathescope," Professors Howard Fehr of Teachers College, E. Glenadine Gibb of Iowa, and Joseph Seidlin of Alfred University. Nearly forty different sectional meetings will provide for all levels and interests.

Division of Fractions with Meaning

Divide: $1 \div \frac{1}{5}$ (by manipulation we have already learned that there are five-fifths in 1 or a whole).

In $8 \div \frac{1}{5}$ there are 8 wholes, so we have 8×5 parts when we divide 8 into "one-fifths."

$8 \div \frac{3}{5}$, in this problem we must first group into fifths, but since we are wanting to know how many groups of three-fifths, or 3 "one-fifths," there are in 8 we need to regroup. In the final grouping there will be $\frac{1}{3}$ as many groups since we want 3 "one-fifths" in each group.

Our process then reads $8 \times 5 \times \frac{1}{3}$ or to put it in a shorter form $8 \times \frac{5}{3}$.

Now we see the divisor ($\frac{3}{5}$) of our problem has been turned upside down.

The arithmetic term for this process is called inversion. We have inverted (turned upside down) the divisor.

Fractions divided by fractions can be explained by the same method. It, however, is a purely mechanical procedure, since fractions cannot be divided by fractions by manipulations.

In the problem $\frac{4}{9} \div \frac{2}{3}$, the fraction $\frac{4}{9}$ represents the quantity to be divided.

To find how many thirds in $\frac{4}{9}$ we have $\frac{4}{9} \times 3$. Finally in regrouping into 2 "one-thirds," the process is $\frac{4}{9} \times 3 \times \frac{1}{2}$ or $\frac{4}{9} \times \frac{3}{2}$ using the reasoning as explained above. The divisor has been inverted in working this problem.

Now proceed to work the example by cancellation.

$$\left(4/9 \div 2/3 = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = 2/3\right)$$

After a bit of drill in each step we are ready to make the generalization that to solve a division-of-fractions problem the divisor is inverted, then proceed to multiply.

Contributed by
Mrs. Jesse Root
Covington, Ohio

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